The Elusive Gains from Nationally Oriented Monetary Policy*

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Abstract

We show that when assessed conditionally on empirically relevant economic developments, the welfare cost of moving away from regimes of explicit or implicit cooperation between two countries for monetary policy may rise to multiple times the cost of economic fluctuations. Divergent economic conditions compound the policymakers' incentives to act non-cooperatively. When international financial markets are incomplete, as captured by restricting financial flows to non-state-contingent bonds, these incentives are driven by the emergence of global imbalances, i.e., large net-foreign-asset positions.

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1 Introduction

The consensus view among academic economists and policymakers is that, once monetary authorities have chartered an optimal course to pursue purely domestic objectives, there is little to be gained from explicit coordination of monetary policy at the international level. We revisit the theory of international monetary cooperation using the same workhorse open economy macro model that has provided its analytical backbone. The key difference between our analysis and the analysis in previous papers is that we assess systematically how the gains from cooperation depend on and evolve dynamically with prevailing economic conditions. Departing from the literature, we show that international spillovers and gains from cooperation are small when global imbalances remain modest, whereas policymakers may be tempted to pursue purely national objectives more and more forcefully when imbalances grow, leading to more sizable foreign spillovers and larger gains from cooperation.

The economic conditions relevant for our argument depend on which financial and real distortions prevail in the economy. But a common thread is that, following realistic economic developments, the cost of pursuing purely domestic objectives can become so large as to rise to multiple times the cost of business cycles.

The model we use to develop our argument has standard features. The world consists of two countries, each specialized in the production of one good that is traded internationally and is an imperfect substitute for the good produced abroad. Both prices and wages are sticky, creating trade-offs for monetary policy. We consider alternative financial market arrangements across countries, including a limited number of bonds that render financial arrangements incomplete, a complete set of Arrow-Debreu securities, and autarky.[1] Apart from the addition of sticky wages and the broader range of financial market arrangements, our model closely follows Benigno and Benigno (2006), and Corsetti, Dedola, and Leduc (2010). Like these authors, we consider cooperative and non-cooperative equilibria with Ramsey optimal strategies for monetary policy. In our baseline, we assume that prices are sticky in the

[1] The static model in Obstfeld and Rogoff (2002) featured only sticky wages and either financial autarky or a case in which terms of trade movements render financial market arrangements irrelevant, as in Cole and Obstfeld (1991).

producer's currency, which implies full pass-through of exchange rate movements to export prices. In sensitivity analysis, we consider local- and dominant-currency-pricing, which limit the pass-through of exchange rate movements to import prices faced by consumers.[2]

In the model, the incentives to act strategically, in a beggar-thy-neighbor fashion, are rooted in the monopoly power of a country over its terms of trade—the monetary analog of the optimal tariff argument in the trade literature. A classic result in the trade literature is that self-interested national policymakers have an incentive to use tariffs to manipulate the terms of trade, in order to improve the utility from consumption residents can obtain in exchange for their labor effort. Stronger terms of trade allow residents to save on disutility of labor while substituting domestic consumption goods with cheaper imports, see, e.g., Dixit (1985). In equilibrium, a tariff war ends up reducing trade altogether—if countries are symmetric, with no change in international relative prices. In the presence of price (and wage) stickiness, a country's monopoly power on its terms of trade gives monetary policymakers a similar incentive.[3] Monetary and trade policies, however, act on different margins. Monetary authorities can only lower the relative price of their country's imports by implementing policies that move the exchange rate, trading off the benefit of stronger terms of trade with the cost of deviating from full employment and price stability (implying price and wage dispersion). This policy trade-off may be resolved differently depending on the features of the economy. But, with all policymakers facing the same incentive, in a non-cooperative equilibrium, they will move their domestic monetary stance in the same direction. World-wide, their correlated stances will translate into inefficient deflation and output gaps, reducing national and global welfare. By contrast, a regime of cooperative monetary policy that internalizes cross border spillovers (the terms-of-trade externality) allows both countries to sustain a (constrained) efficient allocation.

We offer a novel perspective on these classic results, by conditioning welfare anal-

[2] Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Moller (2019) document the empirical relevance of dominant-currency-pricing.

[3] It is worth noting that a familiar caveat from the trade literature also applies to our model: As the number of countries increases and the size of each country decreases, the monopoly power over the terms of trade wanes and so do the gains from cooperation.

ysis on states of the economy that may be more or less likely according to the model itself. Using our model, we compute numerically the dynamic evolution of the economy under cooperation over a finite but large number of periods, which allows us to characterize the distribution of key macroeconomic variables, such as output, real wages, inflation and net foreign assets. Drawing from this (dynamically endogenous) distribution to characterize the relevant economic conditions, we assess the gains from cooperation relative to non-cooperative behavior.

We show that the relevant variables and trade-offs crucially vary with the structure of international financial markets. When international financial markets are limited to non-state-contingent bonds (our baseline), the critical state variable is the net-foreign-asset position (a case we discuss in detail in an introductory analytical section below): gains from cooperation grow larger when net asset positions widen. Key to this result is that the creditor country has a high consumption profile and correspondingly a low marginal utility of consumption because of the transfer (the interest payments) it receives from the debtor. Along a path with a widening net foreign asset position, a falling marginal utility of consumption makes policymakers increasingly willing to trade consumption for leisure: the creditor has an incentive to implement much stronger contractions. Under strategic interactions, the global efficiency losses grow with the intensity of the creditor's monetary contraction.[4]

In a complete market economy, global debt imbalances do not emerge along the dynamic development of the economy. Nonetheless, there are other variables whose evolution tracks the incentives to adopt strategic monetary measures. Specifically, we show that the gains from cooperation increase with the distance of the real wage in either country from its steady-state value. Intuitively, at any point in time the real wage reflects both the evolution of productivity and the way policies have traded-off inflation and unemployment in response to it. In line with the preceding discussion of the incomplete market economy with only non-state-contingent bonds, high real wages are associated with a low marginal utility of consumption: the higher the real wage, the stronger the temptation for policymakers to take advantage of the terms-of-

[4] Along paths with wider imbalances, the exchange rate of the debtor country depreciates also under cooperation. Absent cooperation, the debtor exchange rate ends up depreciating by more.

trade externality, trading-off consumption for leisure through a monetary contraction. Perfect insurance plays a key role in magnifying the incentives to act strategically, and hence the gains from cooperation: complete market contracts improve the trade-off perceived by strategic policymakers, by channelling financial resources to the country when they implement a contraction. Without insurance, the policy trade-off is less favorable. Indeed, when we repeat the analysis for the case of financial autarky, the distance of real wages from the steady state keeps explaining the gains from cooperation, but the mismatch between cooperative and nationally oriented policies is almost negligible. In all our exercises, drawing transition points from the distribution generated by the model allows us to characterize the distribution of the costs of self-oriented national policies and to show that they can exceed the costs of economic fluctuations.

In our baseline analysis, we follow the literature, contrasting the case of full cooperation with (open-loop) non-cooperative strategies under commitment. As mentioned at the beginning of this introduction, however, national central banks have the mandate to achieve a small set of nationally oriented objectives such as domestic price stability and full resource utilization, not to coordinate their policies in support of economic conditions in foreign countries.[5] We show that, in the workhorse model we use, a regime of non-cooperative flexible inflation targeting can support an equilibrium that is close to the one under a regime of full cooperation, despite its sole focus on domestic inflation and output. But for this very reason, under the economic conditions discussed above, the incentives to deviate from flexible inflation targeting and take advantage of the terms-of-trade externality are just as strong as the incentives to deviate from full cooperation.

The idea that international monetary cooperation can yield substantial benefits has faced deep skepticism. Over the past five decades, one can identify at least three waves of reactions. First, throughout the 1970s, some countries systematically failed to deliver on the agreed action plan, reinforcing the view that cooperative agreements were unavoidably plagued by free riding and incentive-compatibility issues. Second, a fundamental objection was formalized by Rogoff (1985), in the context of the disinfla-

[5] See Svensson (2010a) and Reis (2013) for reviews of mandates for central banks.

tion policies during the 1980s. Rogoff warned that, while cooperation may be effective in internalizing cross-border demand spillovers, it may also reduce the credibility of central banks vis-à-vis the private sector, frustrating disinflation efforts. While these criticisms point to extant problems, a third and overarching theoretical criticism was leveled more recently by Obstfeld and Rogoff (2002). These authors claimed that in modern monetary models, gains from cooperation are negligible, relative to *both* best-practice monetary policy *and* full-fledged Nash equilibrium policy strategies.[6]

These waves of criticism led to the consensus view that there is little to be gained from explicit coordination of monetary policy at the international level, articulated in detail by Svensson (2003) and Svensson (2010b), which characterize flexible inflation targeting as best practice monetary policy followed by many central banks. The international monetary policy compact that follows from this consensus view is encapsulated in the maxim "keep your house in order."[7] Starting in the 1980s, a vast body of research in open economy macro has lent theoretical support to this maxim, suggesting that, from a social welfare point of view, if each country could keep its house in order, the global economy would come arbitrarily close to an equilibrium in which policymakers commit to coordinate their policies optimally.[8]

Our study is related to (and in some cases encompasses) key contributions in the literature which either lend support to, or express criticism of the consensus view.[9] Early on, Canzoneri, Cumby, and Diba (2005) also pointed out that the gains from cooperation can become more sizable than in Obstfeld and Rogoff (2002), if the model is augmented with a non-tradeable sector and sector-specific technology shocks. Nonetheless the gains they report remain negligible relative to the cost of business cycles. The relevance of financial markets in shaping these gains is emphasized by

[6] While Obstfeld and Rogoff (2002) focused on a special version of the workhorse monetary model, the subsequent literature did not overturn their findings.

[7] At different times, the maxim "keep your house in order" was reaffirmed by Jerome Powell, the current chair of the Board of Governors of the Federal Reserve, see Federal Reserve Board (2019), as well by his predecessors Janet Yellen, for instance see The Brookings Insitution (2019), and Ben Bernanke, for instance in Bernanke (2017).

[8] For instance, see Sachs and Oudiz (1984), Taylor (1985), Obstfeld and Rogoff (2002), and Pappa (2004). As noted by Taylor (2013) in relation to the Great Moderation period, "[...] policies were executed under a basic understanding that the outcome would be nearly as good as if countries coordinated their policy choices in a cooperative fashion."

[9] Early contributions include Hamada (1976) and Canzoneri and Henderson (1991).

Rabitsch (2012) and, relatedly, by Banerjee, Devereux, and Lombardo (2016), who specifically focus on the role of financial frictions. In none of these studies, however, the welfare analysis is conditional on dynamic economic developments.

Related papers include the work of Korinek (2017), who presents a First Welfare Theorem for open economies, spelling out general conditions on the interactions between policymakers, policy instruments and financial markets that need to be violated to open up any role for cooperation; and Benigno and Benigno (2006), who emphasize that the conditions under which nationally oriented policies have no costs are actually quite restrictive—without however highlighting the importance of the dynamic evolution of these conditions and without pursuing a quantitative analysis. Our analysis complements these earlier studies in this respect. Finally, Benigno (2009) derives a second-order approximation to the welfare function under a cooperative equilibrium and assesses it against policies that completely stabilize inflation in each country. However, since Benigno (2009) does not consider analogous approximations for the welfare function under Ramsey-type non-cooperative policies, it is silent on the gains from cooperation.

The rest of the paper proceeds as follows. Section 2 outlines the model, and the cooperative, non-cooperative policies, including flexible inflation targeting. Section 3 relies on a simplified version of the model to provide insights on the spillovers and externalities giving rise to gains from cooperation in the workhorse monetary model, revisiting classical results in the literature and discussing how these relate to our main contribution. Section 4 lays out our quantitative methods. Section 5 presents our results under incomplete-markets setup with a symmetric portfolio of international bonds. It sizes the welfare gains from cooperation, the cost of business cycles, the incentives to deviate from cooperative strategies, the Pareto efficiency gains, and the incentives to deviate from policies that are consistent with implicit cooperation. Section 6 looks into the role of the currency denomination of bonds and export prices, accounting for incomplete exchange-rate pass-through and the implications of a "dominant currency" in international markets for assets and goods. Section 7 highlights alternative economic conditions that shape the gains from cooperation under complete financial markets or under financial autarky. Section 8 concludes.

2 A Workhorse Open-economy Monetary Model

The analysis builds on a standard two-country, two-goods New Keynesian model similar to those in Obstfeld and Rogoff (2002), Clarida, Gali, and Gertler (2002), Benigno and Benigno (2006), and Corsetti, Dedola, and Leduc (2010). As the model is well known, we present its main features in a compact manner and leave more space for the different monetary regimes under which we conduct our analysis.

2.1 Model Setup

A continuum of agents of mass 1 lives in each of two equally sized countries. In the baseline model, exports are denominated in the currency of the exporting country (producer-currency-pricing), prices and wages are sticky as in Calvo (1983), international financial markets are incomplete as flows are limited to non-state-contingent bonds. Apart from supply side shocks to productivity, our analysis also considers shocks on the demand side, in the form of shocks that alter the time preferences of households (valuation shocks).[10] In the following brief description of the model, given the symmetry of the setup, we focus on country 1, the home country. Appendix A offers more details on the model and describes the extensions we study as sensitivity analysis, including local-currency-pricing, dominant-currency-pricing, complete international financial markets, and financial autarky.

2.1.1 Households

The intertemporal preferences of the representative household in country 1, the home country, are

$$\mathcal{U}_{1,t} = E_t \sum_{j=0}^{\infty} \iota_{1,t+j} \beta^j U_{1,t+j}, \quad (1)$$

$$\text{where } U_{1,t+j} = \ln\left(C_{1,t+j} - \kappa C_{1,t+j-1}\right) - \frac{\chi_0}{1+\chi} L_{1,t+j}^{1+\chi}. \quad (2)$$

[10] An important difference between supply and demand shocks is that the former induce terms of trade movements that facilitate risk sharing even without financial markets and hence rein in the need for international borrowing and lending.

The felicity function, $U_{1,t}$, depends on current and lagged consumption, $C_{1,t}$, as well as hours worked, $L_{1,t}$. In line with the New Keynesian literature, the economy is cashless and abstracts from the utility component of money. Households discount future utility according to $\iota_{1,t+j}\beta^j$; the valuation shock, $\iota_{1,t+j}$, alters the effective time preference of households, capturing households' time-varying preferences for consuming or saving. In an open economy setting, valuation shocks induce international borrowing and lending and generate external imbalances. Following Albuquerque, Eichenbaum, Luo, and Rebelo (2016), we assume the growth rate of $\iota_{1,t}$ to follow an auto-regressive process of order 1

$$\ln\left(\frac{\iota_{1,t}}{\iota_{1,t-1}}\right) = \rho^{\iota}\ln\left(\frac{\iota_{1,t-1}}{\iota_{1,t-2}}\right) + \sigma^{\iota}\varepsilon^{\iota}_{1,t}. \tag{3}$$

The household maximizes intertemporal utility given the budget constraint

$$\begin{aligned} &P^c_{1,t}C_{1,t} + \frac{1}{\phi^b_{1,t}}\left\{P^b_{1,t}B_{11,t} + e_{1,t}P^b_{2,t}B_{12,t}\right\} + \int_S P^D_{1,t+1|t}D_{1,t+1|t} \\ &= W_{1,t}L_{1,t} + B_{11,t-1} + e_{1,t}B_{12,t-1} + D_{1,t|t-1} + \Psi_{1,t}. \end{aligned} \tag{4}$$

The difference between nominal consumption expenditures, $P^c_{1,t}C_{1,t}$, and nominal wage and non-wage income, $W_{1,t}L_{1,t}$ and $\Psi_{1,t}$ respectively, is accounted for by trade in and holdings of financial assets. In detail, households have access to state-contingent bonds $D_{1,t+1|t}$ that only trade within the country at price $P^D_{1,t+1|t}$.

In addition, households take asset positions in international financial markets.[11] We assume that debt is issued in a basket of bonds denominated in different currencies. The cost of acquiring a foreign asset position is thus given by $P^b_{1,t}B_{11,t} + e_{1,t}P^b_{2,t}B_{12,t}$, where $B_{1i,t}$ with $i = \{1, 2\}$ are country 1's holdings of the bond with a price $P^b_{i,t}$ that will pay one unit of the currency of country i the next period. The nominal exchange rate, $e_{1,t}$, converts prices denoted in country 2's currency into country 1's currency. Households take as given a small intermediation cost $\phi^b_{1,t}$. This cost

[11] The typical setup in the open economy macro literature skirts the problem of solving for the portfolio allocation of bonds denominated in different currencies by only tracking net positions and assuming that only one non-state contingent bond is traded. In this framework, assuming that one country can borrow/lend in its own currency introduces an asymmetry playing to the advantage of that country, even if the two countries were modelled as the mirror image of each other in every other dimension. Rather than committing to this assumption, we adopt a framework that still tracks net positions only, but has the flexibility to switch on and off this asymmetry.

is a function of the foreign asset position relative to the size of the economy and ensures stationarity of the distribution of the foreign asset position. The maturing net-foreign-asset (NFA) position is $B_{11,t-1} + e_{1,t} B_{12,t-1}$.

We require the composition of the basket of bonds to satisfy:

$$\eta B_{11,t} = (1-\eta) e_{1,t} B_{12,t}. \tag{5}$$

Two specific choices of the weight parameter η are of interest for the analysis that follows. When $\eta = 0$ (considered in Section 6), country 1 borrows and lends entirely in its own currency, so that its policymakers can manipulate the real value of the country's net asset position by affecting domestic prices. This is an asymmetric "privilege" relative to the other country, where policymakers can affect the returns on its foreign asset position only through exchange rate movements.[12] When $\eta = 0.5$ (our baseline), the net foreign asset position consists of an equally weighted portfolio of bonds denominated in the home and foreign currencies. In this case, neither country enjoys the privilege just described.

Perfectly competitive distributors assemble the final consumption basket, $C_{1,t}$, from the home and (imported) foreign manufactured goods, $C^d_{1,t}$ and $M_{1,t}$, respectively. The distributors solve the cost minimization problem

$$\begin{aligned} &\min_{C^d_{1,t}, M_{1,t}} \; P^d_{1,t} C^d_{1,t} + P^m_{1,t} M_{1,t} \\ &s.t. \\ &C_{1,t} = \left((\omega^c)^{\frac{\rho^c}{1+\rho^c}} \left(C^d_{1,t} \right)^{\frac{1}{1+\rho^c}} + (1-\omega^c)^{\frac{\rho^c}{1+\rho^c}} \left(M_{1,t} \right)^{\frac{1}{1+\rho^c}} \right)^{1+\rho^c}, \end{aligned} \tag{6}$$

where the price of the imported good, $P^m_{1,t}$, equals its price in the foreign country times the nominal exchange rate, $e_t P^d_{2,t}$, under producer-currency-pricing. In this case, the terms of trade for country 1—the price of imports divided by the price of exports—satisfies $\delta_{1,t} = \frac{e_t P^d_{2,t}}{P^d_{1,t}}$. The terms of trade improves if the export price rises relative to the import price, i.e., $\delta_{1,t}$ falls.

[12] For $\eta = 0$, our framework nests the one-bond setup that is typical in the open economy macro literature. Note that when solving a linear approximation of the model around a net foreign asset position equal to 0, the solution cannot capture the effects of exchange rate movements on the value of the maturing position, even when those net foreign asset positions open up dynamically in response to shocks.

2.1.2 Price and Wage Phillips Curves

Households supply $L_{1,t}$ units of labor services to labor unions. The unions, indexed by h, introduce distinguishing characteristics to household labor to produce $L_{1,t}(h)$, before selling it to labor bundlers as in Erceg, Henderson, and Levin (2000) where $\int L_{1,t}(h)dh = L_{1,t}$. The bundlers are perfectly competitive and combine the labor services from the unions into the labor service $L^d_{1,t}$ according to $L^d_{1,t} = \left[\int_0^1 L_{1,t}(h)^{\frac{1}{1+\theta^w}} dh\right]^{1+\theta^w}$. They sell these services at wage $W_{1,t}$ to intermediate goods producers.

The monopolistically competitive unions take the real wage desired by households, $\tilde{W}_{1,t}/P_{1,t}$, which equals the marginal rate of substitution between the disutility of labor and consumption, as the cost of labor and set nominal wages as in Calvo (1983). Each period, with probability $1-\xi^w$, a union gets to adjust its wage $W_{1,t}(h)$ optimally; otherwise, a union adjusts its wage by the steady-state inflation rate, $\bar{\Pi}$. Union h solves

$$\max_{W_{1,t}(h)\{L_{1,t+j}(i)\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\xi^w)^j \Lambda_{1,t+j} \left[(1+\tau^w)\bar{\Pi}^j W_{1,t}(h) - \tilde{W}_{1,t+j}\right] L_{1,t+j}(h)$$

$$s.t.$$

$$L_{1,t+j}(h) = \left[\frac{W_{1,t+j}(h)}{W_{1,t+j}}\right]^{-\frac{1+\theta^w}{\theta^w}} L^d_{1,t+j}, \tag{7}$$

where the stochastic discount factor, $\Lambda_{1,t+j}$, is such that $\Lambda_{1,t+j} = \beta^j \frac{MU_{1,t+j}}{MU_{1,t}} \frac{P^c_{1,t}}{P^c_{1,t+j}}$, and where $MU_{1,t}$ is the marginal utility of consumption. Equation (7) relates the bundlers' demand for the labor of union h to the union's wage $W_{1,t+j}(h)$. The subsidy τ^w is set to make the level of the labor supply efficient in the steady state.

We model sticky nominal prices analogously. Monopolistically competitive firms produce differentiated varieties using a linear technology

$$Y_{1,t}(i) = \exp(z_{1,t}) L^d_{1,t}(i), \tag{8}$$

where $z_{1,t}$ is the country-wide technology shock. The term $L^d_{1,t}(i)$ is the demand of firm i for the aggregate labor services $L^d_{1,t}$ where $L^d_{1,t} = \int L^d_{1,t}(i)di$. The marginal production costs are therefore $W_{1,t}/\exp(z_{1,t})$. Competitive bundlers combine the varieties into the home manufactured good according to $Y_{1,t}^d = \left[\int_0^1 Y_{1,t}(i)^{\frac{1}{1+\theta^p}}\, di\right]^{1+\theta^p}$ and sell it at the price $P_{1,t}^d$ domestically and at the price $P_{1,t}^d/e_t$ abroad.

Variety producers set nominal prices as in Calvo (1983). Each period, a producer adjusts its price $P_{1,t}$(i) with probability $1-\xi^p$ optimally and with probability ξ^p by the steady-state inflation rate $\bar{\Pi}$. Producer i solves

$$\max_{P_{1,t}(i),\{Y_{1,t+j}(i)\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\xi^p)^j \Lambda_{1,t+j} \left((1+\tau^p)\, \bar{\Pi}^j P_{1,t}(i) - \frac{W_{1,t+j}}{\exp(z_{1,t+j})} \right) Y_{1,t+j}(i)$$

$$s.t.$$

$$Y_{1,t+j}(i) = \left[\frac{P_{1,t+j}(i)}{P_{1,t+j}^d} \right]^{-\frac{1+\theta^p}{\theta^p}} Y_{1,t+j}^d. \tag{9}$$

Equation (9) relates the demand by the bundlers for variety i, $Y_{1,t+j}(i)$, to the price of the variety, $P_{1,t+j}(i)$. The sales subsidy τ^p is set to eliminate the relative price distortions due to monopolistic competition in the deterministic steady state.

2.1.3 Market Clearing

Market clearing for the domestically produced good implies that

$$Y_{1,t}^d = C_{1,t}^d + M_{2,t}, \tag{10}$$

where $M_{2,t}$ denotes the demand of the foreign country for the domestic good. Analogously, market clearing for the good produced abroad requires

$$Y_{2,t}^d = C_{2,t}^d + M_{1,t}. \tag{11}$$

Finally, domestically traded bonds are in zero net supply, requiring $D_{1,t+1|t} = 0$. For internationally traded bonds, market clearing requires

$$B_{11,t} + B_{21,t} = 0, \tag{12}$$

$$B_{12,t} + B_{22,t} = 0. \tag{13}$$

2.2 Monetary Policy

Monetary policymakers in each country set the path of their respective policy instrument, $i_{1,t}$ and $i_{2,t}$, to optimize their assigned objective function subject to the private optimality and market clearing conditions associated with the model as detailed in Appendix A. The private optimality and market clearing conditions are summarized by

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0, \tag{14}$$

where $\tilde{x}_t$ denotes the $(N-2) \times 1$ vector of endogenous variables excluding policy instruments and ζ_t is the vector of the exogenous shocks. The objective functions differ between the cooperative and the non-cooperative policy game, as detailed next.

2.2.1 Cooperative Policies

In the cooperative game, the policymakers maximize global welfare defined as the weighted average of the utility of the representative households in the two countries, $\alpha_1 \mathcal{U}_{1,t} + (1-\alpha_1)\mathcal{U}_{2,t}$, under full commitment

$$\begin{aligned} &\max_{\{\tilde{x}_t, i_{1,t}, i_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + (1-\alpha_1) U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)\right], \\ &s.t. \\ &E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0. \end{aligned} \tag{15}$$

We refer to the monetary policies associated with the cooperative game as "cooperative policies."

2.2.2 Non-cooperative or Nationally Oriented Policies

We model the non-cooperative interactions between policymakers in different countries as an open-loop Nash game. Let $\{i_{j,t,-t^*}\}_{t=0}^{\infty}$ denote the sequence of policy choices by player $j = [1, 2]$ before and after, but not including period t^*. An open-loop Nash equilibrium is a sequence $\left\{i^*_{j,t}\right\}_{t=0}^{\infty}$ with the property that for all t^*, i^*_{j,t^*} maximizes player $j's$ objective function subject to the structural equations of the economy in Equation (14) for given sequences $\left\{i^*_{j,t,-t^*}\right\}_{t=0}^{\infty}$ and $\left\{i^*_{-j,t}\right\}_{t=0}^{\infty}$, where $\left\{i^*_{-j,t}\right\}_{t=0}^{\infty}$ denotes the sequence of policy moves by the other player. Each player's action is the best response to the other players' best responses.

With policymakers needing to specify a complete contingent plan at time 0 for their respective instrument variable, we can recast each player's optimization problem as an optimal control problem given the policies of the other player

$$\begin{aligned}
&\max_{\{\tilde{x}_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t), \\
&s.t. \\
&E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0 \\
&\text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.
\end{aligned} \tag{16}$$

We refer to the monetary policies associated with the non-cooperative game as "nationally oriented policies."

2.2.3 Keep-Your-House-in-Order Policies

The objective function of the policymakers need not coincide with the utility functions of the representative households. Using the general loss function $\mathcal{L}_j$, we modify the non-cooperative game in Section 2.2.2 to be

$$\begin{aligned}
&\max_{\{\tilde{x}_t, i_{j,t}\}_{t=0}^{\infty}} -E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t), \\
&s.t. \\
&E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0 \\
&\text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.
\end{aligned} \tag{17}$$

Following Svensson (2003), we capture flexible inflation targeting with the simple loss function

$$\mathcal{L}_j = w_\pi(\pi_{j,ct}^4 - \bar{\pi}^4)^2 + w_y(y_{j,t}^{gap})^2, \tag{18}$$

where $\pi^4_{j,ct}$ denotes annualized consumption price inflation and $y^{gap}_{j,t}$ the output gap in country j.[13]

3 Analytical Insights From a Simplified Version of the Model

In this section, we provide economic insights on the nature of the spillovers that give rise to gains from cooperation and create incentives for opportunistic behavior. Following the recent open economy macro literature, we show that the relevant cross-country spillovers are rooted in a country's monopoly power over its own terms of trade—mapping the theory of the optimal tariff in trade into the monetary framework of New Keynesian and New-Open-Economy Macroeconomics. We also elucidate the reason why these gains change with the evolution of the economy. We develop these insights using a version of our model which is simplified along two main dimensions.

First, we gain tractability by restricting the parameters to bring our model close to the specification in Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2002). Namely, we impose a unitary trade elasticity; we set $\chi = 0$, which yields quasi-linearity of the utility function with respect to labor; and we set $\tau^p = \theta^p$ and $\tau^w = \theta^w$ to remove distortions from monopolistic competition. Moreover, we set ξ^w to 0, implying that wages are fully flexible and assume complete exchange rate pass-through or producer-currency-pricing. We exclude consumption habits by setting $\kappa = 0$. Finally, we assume symmetry by setting domestic and foreign parameters at identical values. Sovereign debt is issued as bonds denominated in the currency of each of the two countries in equal proportions, i.e., we set $\eta = \frac{1}{2}$.

Second, we posit a finite horizon, assuming that up until period $T-1$ policymakers cooperate. In period T, the last period of the model, we let policymakers reconsider whether to cooperate or not. Essentially, our analysis reduces to a one-period model where policy and agents' decisions depend on the inherited distributions for prices and borrowing/lending in international capital markets. This structure allows us to

[13] We defined the output gap as the difference between output in the model with nominal price and wage rigidities and output in the analogous model with flexible prices and wages.

explore the consequences of different values of the inherited variables without being specific about the underlying economic disturbances that have given rise to these values; in this analytical section, we will abstract from saving and productivity shocks altogether. Regardless of differences in inherited variables, we will first assume that each country's felicity function has an equal weight in the global welfare function. We will revisit this assumption at the end of the section.

With all these assumptions in place, the equilibrium conditions of the model can be expressed as functions of variables set in period $T-1$ leaving real marginal cost in each country, $mc_{1,T}$ and $mc_{2,T}$, as the only choice variables in period T. Marginal costs, as discussed by Corsetti and Pesenti (2005), track the monetary stance in each economy: under sticky prices, a monetary expansion maps into higher demand that drives up nominal wages and, hence, marginal costs (vice versa for a contraction). Therefore, marginal costs can be treated as the choice variables for monetary policymakers.

Leaving the analytical derivations to Appendix B, hereafter, we focus on the three equations defining the analytical core of the model and our argument. The first is the equilibrium terms of trade, which in the simplified model boils down to the following function in real marginal costs and the outstanding debt:

$$\delta_{1,T} = \frac{mc_{1,T} - \frac{1}{\Pi_{1,T}} \frac{\chi_0}{2(1-\omega_1^c)} \bar{B}_{1,T-1}}{mc_{2,T} + \frac{1}{\Pi_{2,T}} \frac{\chi_0}{2(1-\omega_1^c)} \bar{B}_{1,T-1}}. \quad (19)$$

Notice that price inflation ($\Pi_{i,T}$ with $i = \{1, 2\}$) is also a function of real marginal costs only.[14] Absent foreign debt, the equilibrium terms of trade (like the exchange rate) must equal the ratio of the monetary stance in the two countries, i.e., $\delta_{1,T} = \frac{mc_{1,T}}{mc_{2,T}}$. It is easy to see that a monetary contraction in country 1 (reducing $mc_{1,T}$) makes the country's exports relatively more expensive: the terms of trade of country 1 improve. By the same token, holding constant marginal costs, an outstanding stock of debt for country 1 ($\bar{B}_{1,T-1} < 0$) implies that, in equilibrium the terms of trade worsen. For intuition on these results, consider that the interest payments on the debt amount to a transfer of income from the debtor to the creditor country. As is well understood, with home bias in consumption, this transfer results in a drop in

[14] The exact relationship is $\Pi_{i,T} = \left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{i,T}^{-\frac{1}{\theta^p}} \right)^{\theta^p}$, where we set the steady state inflation rate $\bar{\Pi} = 1$.

the global demand for the goods produced by the debtor (the creditor will use the income disproportionately to buy own goods), causing the international price of the debtor's output to fall.

The other two expressions are the indirect utility functions of the households in country 1 and its symmetric counterpart in country 2. Omitting the latter to save space, we have:

$$\begin{aligned} U_{1,T} &= \ln\left(\frac{1}{\chi_0}\right) + \ln\left(mc_{1,T}\right) - \left(1-\omega_1^c\right)\ln\left(\delta_{1,T}\right) \\ &- \omega_1^c \Delta_{1,T}^p mc_{1,T} - \delta_{1,T}\left(1-\omega_1^c\right)\Delta_{1,T}^p mc_{2,T}. \end{aligned} \quad (20)$$

The term $\Delta_{1,T}^p$ denotes price dispersion which, like inflation, is only a function of real marginal costs $mc_{1,T}$.[15] This expression establishes two crucial features of the monetary transmission in the workhorse open-economy model. First, monetary policy in country 2 has spillovers in country 1 via terms of trade movements (directly as well as indirectly, interacted with price dispersion), proportionally to openness. Specifically, it is easy to see that a monetary contraction in country 2 that weakens the terms of trade of country 1 ($\delta_{1,T}$ rises) lowers welfare in this latter country, because its residents will have to produce more output to maintain any given level of consumption. Second, starting from the symmetric cooperative equilibrium (or the policy choices in the closed economy) with $mc_{1,T} = mc_{2,T} = 1$, country 1 can improve its welfare via a monetary contraction that, at the margin, strengthens its own terms of trade. This is apparent when evaluating the expression in 19 for $\bar{B}_{1,T-1} = 0$ and $mc_{2,T} = 1$, where the relevant terms in the welfare function simplify to $\omega_1^c \ln\left(mc_{1,T}\right) - \Delta_{1,T}^p mc_{1,T}$. Because of the effect of a monetary contraction on domestic prices, however, the incentive to resort to it will be moderated by the inefficient (and thus, welfare reducing) price dispersion a monetary tightening would imply.

[15] As derived in the appendix, the condition determining price dispersion with respect to marginal cost for country i is: $\Delta_{i,t}^p = (1-\xi^p) mc_{i,T}^{-\frac{1+\theta^p}{\theta^p}} + \xi^p \left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{i,T}^{-\frac{1}{\theta^p}}\right)^{1+\theta^p} \Delta_{i,t-1}^p$, where $i \in \{1,2\}$.

3.1 Cooperative Policies

By virtue of our simplifying assumptions, we can obtain a tractable analytical characterization of the optimal policy under cooperation. As shown in the appendix, the optimal cooperative monetary stance for country 1 can be expressed as

$$mc_{1,T} - 1 = \frac{\chi_0}{1 + \Delta_1^{p''}} \bar{B}_{1,T-1} \tag{21}$$

and symmetrically for country 2. $\bar{B}_{1,T-1}$ is the outstanding net foreign asset of country 1, positive if the country is the creditor and negative if it is the debtor country; $\Delta_1^{p''} = \frac{1+\theta^p}{\theta^p} \frac{1-\xi^p}{\xi^p}$ is the second derivative of the condition determining price dispersion with respect to marginal cost for country 1, evaluated around the no-debt cooperative equilibrium with no inherited dispersion, $\Delta_{1,T-1} = 1$. As shown in the appendix, inherited nonzero price dispersion does not alter our conclusions.

As starting point for our analysis, we determine the direction in which outstanding debt moves the optimal cooperative monetary stance in the debtor and the creditor country, relative to a no-debt baseline.

Proposition 1 *In the absence of outstanding net foreign assets, the optimal cooperative monetary stance sets marginal costs equal to 1, so that the terms of trade are also equal to 1 (i.e., $\delta_{1,T} = 1$). Compared to this equilibrium, if the outstanding net foreign assets are nonzero, the monetary stance is tighter in the debtor country (the real marginal cost is below 1), and looser in the creditor country (the real marginal cost is above 1).*

Leaving the detailed proof for Section B.1 of the appendix, the first part of the proposition restates a well-known result by Corsetti and Pesenti (2005). Without debt outstanding, the allocation is symmetric, with the monetary stance keeping the output gap and inflation (price dispersion) equal to zero in both countries. Against this benchmark, the second part of the proposition illustrates the asymmetry created by outstanding debt. Holding monetary policy (suboptimally) constant with real marginal costs equal to 1, the repayment of debt in period T reduces consumption in the debtor country for any level of leisure. This is due to the combination of the direct effect on relative income of the interest payment (a resource transfer) for given international prices; and the indirect effects of the exchange rate equilibrium adjustment—we have seen above that the terms of trade of the debtor deteriorates. By internalizing the cross-border monetary spillovers, cooperative monetary authorities trade off output gap and inflation stability across the two countries, to make the world allocation less asymmetric. A relative expansionary stance of the creditor contains the adverse movements of international prices against the debtor. At the same time, at the margin, it boosts the creditor's demand for the debtor's good, facilitating repayment at improved terms of trade for the debtor.

3.2 Non-Cooperative Policies

Underlying our analysis is a classical result in modern open economy macro. Acting strategically, each policymaker will have an incentive to move the terms of trade in their own favor with a monetary contraction, in an attempt to improve their own social welfare at the expense of welfare in the other country. At the margin, the incentive is to save on labor efforts, without suffering a large fall in overall consumption as households replace domestically produced goods with cheaper imports. This result is best appreciated in our simplified model, where, up to a first-order approximation, the non-cooperative policy stance for country 1 can be characterized as follows:

$$mc_{1,T} = 1 - \frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}} - \frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} \bar{B}_{1,T-1}. \tag{22}$$

Outstanding net foreign assets affect the incentives for policymakers to manipulate the terms of trade. As shown in Equation 22, the equilibrium monetary contraction will be different for debtor and creditor nations. It is the creditor that has the upper hand in the game, as stated in the following proposition and as proved in Section B.2 of the appendix.

Proposition 2 *Relative to the cooperative equilibrium, in the non-cooperative equilibrium the monetary stance is contractionary in both countries but remains symmetric in the absence of debt. Debt induces an asymmetry in the non-cooperative monetary* *stance: the creditor's (debtor's) terms of trade improve (worsen) and the improvement (worsening) depends on the size of the net-foreign-asset position.*

Without debt, the terms of trade is still 1, as under the cooperative equilibrium. This result is in line with Obstfeld and Rogoff (2002). With debt, the relative monetary stance, creditor-to-debtor, goes in the opposite direction relative to the cooperative case—i.e., the coefficient on the term $\bar{B}_{1,T-1}$ enters with a negative sign in Equation 22 and a positive sign in Equation 21. Essentially, the strategic monetary contraction is stronger in the creditor than in the debtor country.

In equilibrium, despite this asymmetry, the policymakers' strategic contractions will largely neutralize each other in their effects on the exchange rate. However, the global contraction will open a world output gap where employment and consumption are (at different rates) inefficiently low. We analyze the consequences on welfare next.[16]

3.3 Assessing the Gains from Cooperation

To map the results presented so far into an analytical assessment of the gains from cooperation, we derive a second-order approximation of the global welfare function, and evaluate it at the cooperative and the non-cooperative equilibrium choices of real marginal costs. We start by characterizing the global welfare function in the following proposition, leaving the proof to Section B.3 of the appendix.

Proposition 3 *The (purely) quadratic approximation to the global welfare function around the symmetric cooperative equilibrium with zero debt (and* $\Delta_{1,T-1} = \Delta_{2,T-1} = 1$*) is given by*

$$U_T - \bar{U} \approx -\frac{1}{4}\left(1 + \Delta_1^{p''}\right)(mc_{1,T} - 1)^2 - \frac{1}{4}\left(1 + \Delta_2^{p''}\right)(mc_{2,T} - 1)^2$$

[16] While in the workhorse open economy monetary model each country has an incentive to *appreciate* its currency in real terms, in the two-country two-sector model by Bergin and Corsetti (2020), strategic monetary authorities seek to *depreciate* their currency—a result that is more closely aligned with the popular idea that competitive devaluations are prevalent. The difference resonates with the debate in the trade literature contrasting the optimal tariff argument with the argument in favor of using trade policy to enhance competitiveness, see, for instance, Ossa (2014). However, as long as the magnitude of cross-border spillovers change with the dynamic evolution of the economy, the main conclusions of our paper apply to any class of open economy models.

$$+\frac{\chi_0}{2}\left(mc_{1,T}-1\right)\bar{B}_{1,T-1}-\frac{\chi_0}{2}\left(mc_{2,T}-1\right)\bar{B}_{1,T-1}-\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2. \tag{23}$$

As shown in Section B.4 of the appendix, substituting the cooperative and the non-cooperative polices into Equation 23, we obtain our main result, which is stated in the next proposition and proved in Section B.4 of the appendix.

Proposition 4 *The (purely) quadratic approximation to the welfare gains from cooperation are increasing in the size of the net foreign asset position:*

$$DDU_T^{co,nc} = \frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0}{1+\Delta_1^{p''}}-\frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_+^c\Delta_1^{p''}}\right)^2\bar{B}_{1,T-1}^2>0 \tag{24}$$

where $DDU_T^{co,nc}=\left(U_T^{co}-\bar{U}\right)-\left(U_T^{nc}-\bar{U}^*\right)$ *is the second-order approximation of the difference in the change of global welfare from the steady state.*

As we have established, in the non-cooperative equilibrium, both policymakers give in to the incentive to pursue contractionary policies, but, with outstanding debt, it is the creditor country that runs a tighter monetary policy. This asymmetry results into the inefficient strengthening of the creditor's terms of trade. Our new proposition shows that this asymmetry also adds to the reduction in global welfare from strategic behavior.

In the appendix, we elaborate on the economics of this result.[17] A country with a positive asset position enjoys more consumption, which compresses the marginal utility of consumption and boosts the incentive to trade off consumption for leisure. Higher external imbalances translate into deeper strategic contractions by the creditor. In equilibrium, higher external imbalances drive the allocations further away from their (constrained) efficient counterparts.

An argument often made by the literature is that the cooperative equilibrium is arbitrarily close to an equilibrium where policymakers "keep their own house in order" by pursuing a policy of price stability. In this regime, for the case of strict inflation

[17] See Section B.6 of the appendix.

targeting, policymakers set $\Pi_{1,T} = \bar{\Pi}$ and $\Pi_{2,T} = \bar{\Pi}$ and thus $mc_{1,T} = mc_{2,T} = 1$ regardless of the net foreign asset position. These insights and Proposition 3 lead us to our next proposition, proved in Section B.5 of the appendix.

Proposition 5 *While global welfare is lower under strict inflation targeting than under the optimized cooperative policies, it is higher than under non-cooperative policies. To a second-order approximation, the disadvantage (advantage) vis-à-vis the cooperative (non-cooperative) policies is increasing in the net foreign asset position.*

Under an inflation targeting regime, the policymakers do not internalize cross-border monetary spillovers, but do not act strategically in a beggar-thy-neighbor manner. Hence, in welfare terms, the resulting equilibrium is not far from full cooperation.

3.4 Generalization: State-Contingent Welfare Weights

Thus far, we have measured the global gains from cooperation under the assumption that the utility of each country has an equal weight in the global welfare function, i.e. $\alpha_1 = \frac{1}{2}$, reflecting the symmetry of the two countries in the initial steady state. We pursue two alternative approaches to pick state-contingent weights in period T, when the gains from cooperation are assessed: the Pareto approach and the Negishi approach. Under the Pareto approach, we adjust the weights in the global welfare function so that both countries are at least as well off in the cooperative equilibrium as they would be in the non-cooperative equilibrium. Under the Negishi approach, the weights reflect the marginal utilities of wealth in the non-cooperative equilibrium in each country. A series of propositions in Appendix C shows that under both state-contingent approaches cooperation is superior to non-cooperation. Furthermore, the gains from cooperation increase quadratically in the size of the net foreign asset position, just as under the scheme with symmetric welfare weights.

4 Quantitative Analysis

In the preceding section, we used a simplified version of our model to illustrate analytically the macroeconomic and welfare differences between cooperative and non-cooperative arrangements, and how and why the incentives to deviate from cooperation change with economic conditions, specifically, with net foreign assets. For analytical tractability, apart from a number of parametric restrictions, we took the state variables of the model as given—without imposing that their predetermined values should also be consistent with the structure of the model—and assumed a finite horizon. In the rest of the paper, we relax all these assumptions and rely on numerical simulations to quantify the gains from cooperation. We will let the values of the predetermined variables depend on the sources of economic fluctuations, the histories of exogenous shocks, and, most crucially, on how policies resolved trade-offs over time. We will contrast optimal cooperative and non-cooperative policies, and substantiate that the gains from cooperation are tightly linked to the evolution of the net-foreign-asset position.

Despite the added complexity of the full model described in Section 2, the economic intuition highlighted in the five propositions of Section 3 provide tight guidance for the interpretation of our quantitative results.

4.1 Parameterization and Solution Method

The model parameters are reported in Table 1. While most of the parameters in this table are standard in the literature, it is important to note that there is no general agreement on the appropriate value of the trade elasticity of substitution for aggregate open economy models. Some authors have emphasized elasticities well above one as empirically relevant. For instance, Bernard, Eaton, Jensen, and Kortum (2003) report a trade elasticity of substitution in the range of 4, while Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008) stress that values lower than 1 can also be empirically relevant. Accordingly, we explore the whole range of relevant values, $[0.65, 4]$, as measured by $\frac{1+\rho^c}{\rho^c}$, the trade elasticity in our model. We assume technology shocks to be the only source of disturbances in this section; we bring the valuation shock back into the analysis in Section 6.

We use second-order perturbation methods to approximate the conditions for an equilibrium implied under cooperative and nationally oriented policies (see the maximization problems in Equations 15 and 16, respectively). To derive the analytical conditions for an equilibrium under these two policies we apply the symbolic differentiation toolbox of Bodenstein, Guerrieri, and LaBriola (2019). We follow Benigno and Benigno (2006) in using domestic price inflation as the policy instrument.[18]

Table 1: Parameters for the Baseline Two-Country Model

Parameter	Used to Determine	Parameter	Used to Determine
$\beta = 0.995$	discount factor	$\kappa = 0.5$	consumption habits
$\chi = 1/2.84$	labor supply elasticity $= \frac{1}{\chi}$	$\bar{L} = 1/3$	steady-state labor supply to fix χ_0
$\xi^p = 0.75$	price stickiness	$\xi^w = 0.75$	wage stickiness
$\theta_p = 0.1$	price markup (before subsidy)	$\theta_w = 0.1$	wage markup (before subsidy)
$\tau_p = 0.1$	subsidy to producers	$\tau_w = 0.1$	subsidy to unions
$\omega^c = 0.88$	home bias in consumption	$\alpha_1 = 0.5$	weight of home country in global welfare
$\phi^b = 10^{-4}$	governs bond intermediation cost	$\eta = 0.5$	share of bonds in home country currency
$\rho^z = 0.95$	persistence of tech. shock	$\sigma^z = 0.015$	std. of tech. shock

Note: This table summarizes the parameterization of the baseline two-country model described in Section 5 at quarterly frequency.

4.2 Assessing the Gains from Cooperation and the Incentive to Deviate from Cooperative Behavior

Hereafter, we define the way we size the gains from cooperation, the cost of business cycles, the incentives to deviate from cooperation, Pareto efficiency gains, and the incentives to deviate from implicit cooperative arrangements implied by inflation-targeting policies.

[18] For the second-order perturbation solution we rely on Dynare. See Adjemian, Bastani, Karamé, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011). All the model statistics reported below are computed using a true second-order approximation, using the pruning algorithm described in Kim, Kim, Schaumburg, and Sims (2008).

4.2.1 The Gains from Cooperation

We size the gains from cooperation, relying on a comparison of the *conditional* welfare values attained under the cooperative and the nationally oriented policies.[19] Specifically, rather than focusing on one arbitrary point (which in the literature is typically the deterministic steady state), we sample transition points from the ergodic distribution of the model under the cooperative equilibrium, and assess welfare conditional on each of these points. To this purpose, we first draw random sequences of shocks for 250 periods. The final point in this series provides the transition point for the welfare comparison of the two policies (denoted by $\tilde{x}_{250}$). We then compare the conditional welfare implied by non-cooperative policies starting in period 251 with the conditional welfare implied by continued reliance on cooperative policies. We construct a distribution of gains from cooperation (losses from non-cooperative policies) based on a sample of 1000 transition points from the ergodic distribution.

We measure the units of conditional welfare using the standard metric of consumption equivalent variation. We report the consumption subsidy that would have to be offered in perpetuity to households for them to attain the same level of welfare under the nationally oriented policies as under the cooperative policies. The subsidy net rate τ equals

$$\tau = \exp\left(\frac{1-\beta}{\alpha_1}\left(Welf_t^{co} - Welf_t^{nc}\right)\right) - 1.$$

$Welf_t^{co} = \alpha_1 \mathcal{U}_{1,t}^{co} + (1-\alpha_1)\mathcal{U}_{2,t}^{co}$ denotes the global welfare level attained under the cooperative equilibrium and, similarly, $Welf_t^{nc}$ is the global welfare level attained under the non-cooperative equilibrium. The derivation of this subsidy is provided in Appendix D. In the following sections, we characterize the distribution of this subsidy across transition points by reporting its mean, as well as the fifth and ninety-fifth percentiles.

[19] See Kim and Kim (2018) for a discussion of how optimal policies based on conditional welfare measures can appear suboptimal when ranked with unconditional welfare measures.

4.2.2 The Cost of Business Cycles

To interpret the gains from cooperation, τ, we compare them against a measure of the cost of economic fluctuations. Focusing on the cooperative equilibrium, following Lucas (2003), we size the cost of economic fluctuations as the consumption equivalent variation that, starting from the deterministic steady state, with all current and future shocks excluded, would keep households indifferent from having to face shocks.[20]

4.2.3 Incentives to Deviate

As the state of the economy varies across points of the ergodic distribution, there could be substantial variation in the incentives for policymakers to keep their commitment to cooperation, as opposed to considering a new course of policy strategies, more narrowly focused on national objectives. To assess these incentives, we consider the following two-stage game.

In the first stage, conditional on each transition point $\tilde{x}_{250}$, we let a country choose between *cooperate* or *deviate*. If country j chooses *cooperate*, its objective is the global welfare function $\alpha_1 \mathcal{U}_{1,t} + (1 - \alpha_1)\mathcal{U}_{2,t}$; if it chooses *deviate*, its objective is the national welfare function $\mathcal{U}_{j,t}$, for $j = 1, 2$. In the second stage, the two countries play an open-loop Nash game (as described in Section 2.2.2) that determines each country's welfare given the actions selected in the first stage and conditional on the transition point $\tilde{x}_{250}$.[21] Note that, if both countries choose *cooperate* in the first stage, the second stage game yields the same outcomes as the cooperative policies defined in Section 2.2.1. Analogously, if both countries choose deviate in the first stage, the second-stage game yields the same outcomes as the non-cooperative policies defined in Section 2.2.2.

[20] Mechanically, the two economies have identical second-order perturbation solutions, but for a vector of constants (the stochastic shift factor) that enters the economy with shocks and that drops out of the other economy without shocks. To encompass the effects of current shocks, we draw 1000 random shock vectors, and average the consumption equivalent variation for each shock vector.

[21] The objective functions of the policymakers in Equation (16) are determined by the actions chosen in the first stage of the game.

4.2.4 Efficiency Gains and the Pareto Frontier

We complete our welfare analysis by sizing the efficiency gains of cooperation. We construct the Pareto frontier by varying the welfare weight ω over the range from 0 to 1 at each transition point. We compare the non-cooperative and cooperative allocations by considering the changes in utility consistent with making either country better off without making the other country worse off. Previewing our results, we find that efficiency gains play the lion share in our baseline, and remain significant also when one of the country benefits from the privilege of issuing the dominant currency.

5 Fragility of Cooperation with Growing External Imbalances

Under the incomplete international financial markets setup that limits flows to occur via non-state-contingent bonds, consumption smoothing in the face of economic disturbances leads to the accumulation of external imbalances (NFA positions). We have seen that, in our standard open economy model, policymakers always have an incentive to manipulate the terms of trade strategically. Once external imbalances widen, non-cooperative policies become asymmetric for debtors and creditors, and this asymmetry amplifies the inefficiency and welfare losses from deviating from cooperation. Below, we elaborate on this result using our full model.

5.1 Net Foreign Assets and the Gains from Cooperation

Figure 1 provides a striking illustration of the importance of external balance accumulation as a key driver of the gains from cooperation, in line with our analytical results and Proposition 4. To facilitate comparability with much of the literature, the results shown in the figure refer to an economy for which technology shocks are the only economic disturbances. The top panel depicts the ergodic probability density function (PDF) for the NFA position of the home country, measured in percent of annualized output for four values of the trade elasticity $\frac{1+\rho^c}{\rho^c} = \{0.7; 0.8; 2; 4\}$. The bottom panel plots the gains from cooperation against NFA positions for each of the 1000 transition points drawn from the ergodic distribution under the cooperative policies, as detailed in Section 4.2.

In our baseline, assuming only technology shocks implies that the (ergodic) distribution of the NFA positions under the cooperative policies varies with the trade elasticity non-monotonically. As explained by Cole and Obstfeld (1991), technology shocks cause no accumulation of net foreign assets/debts under a unitary trade elasticity and (if the home bias is symmetric) log-utility over consumption (implying that home and foreign goods are neither substitutes nor complements). In this limit case, the terms of trade movements provide efficient risk sharing without financial assets. The distribution of NFA positions in Figure 1, however, becomes more dispersed either as the trade elasticity falls below 1 or as it rises above 1. Extreme NFA positions are more likely under a high trade elasticity, well above 1, than under a low elasticity, well below 1.

The gains from cooperation depend on the NFA position at that transition point. As shown in the bottom panel of Figure 1, fixing the value of the trade elasticity, the gains from cooperation increase with the (absolute) value of the NFA position. Conversely, fixing the value of the NFA position, the gains from cooperation increase as the value of the trade elasticity declines. For NFA positions close to zero, the gains are negligible regardless of the value of the trade elasticity.

The figure suggests that the other endogenous variables, apart from the NFA position, play a negligible role in influencing the size of the gains. To wit, if other variables played a sizable role, the points shown in the bottom panel would not line up in a neat parabola. This result is in line with the analytical derivations in our simplified model.[22] Moreover, we confirmed it by regressing the gains from cooperation on the NFA positions and their squares. The regression yields an R^2 statistic varying from 0.94 to 0.97 depending on the trade elasticity of substitution (in the range from 0.65 to 4). This tight fit implies that the values of other endogenous variables at the transition point play no meaningful role in the gains from cooperation

[22] For the simplified model underlying our analytical results, we can show that the values of the technology shocks have no first-order effects on the gains from cooperation. Technology shocks matter only indirectly through their impact on the NFA positions. Similarly, the dispersion of prices at the transition point is not the driving force behind the gains from cooperation.

independently of the NFA positions.

Figure 2 shows the welfare gains from continuing to cooperate rather than adopting nationally oriented policies, averaging over the 1000 transition points (the dashed-dotted line), against the trade elasticity. The figure shows the mean together with the 5^{th} and 95^{th} percentiles (the dotted lines) of the distribution of gains associated with the 1000 transition points. For comparison, the figure also plots the cost of business cycles, i.e., the gains that would accrue if all fluctuations were to be eliminated, shown by the dashed line.

The figure highlights a key result. The gains from cooperation can be much higher than the cost of business cycles—and are large for trade elasticities different from 1. With trade elasticities higher than 1, this result is driven by the higher likelihood of large trade imbalances. Conversely, for trade elasticities lower than 1, the gains from cooperation are large even for modest trade imbalances—this is because the equilibrium response of the terms of trade to monetary policy is quite pronounced, causing large monetary spillovers. Note that the distance between the percentiles shown in the graph implies that the variation in the gains is higher when the average gains from cooperation are higher. Finally, when the trade elasticity is near 1, the gains are negligible. As we have seen, in that case, with only technology shocks, the net foreign asset position is concentrated at 0.

5.2 Incentives to Deviate from Cooperation and the Distribution of Gains and Losses

The outcomes discussed in the preceding section for each particular transition point incorporate the reaction of the foreign country to the non-cooperative policy switch in the home country. Accordingly, we only captured the final equilibrium. To account for the incentives to deviate from the cooperative behavior, we rely on the two-stage game described in Section 4.2.3. At each transition point, in the first stage, we let a country choose between *cooperate* or *deviate*; in the second stage, we let them play an open-loop Nash game conditional on the choices in the first stage.

For all combinations of actions in the first stage of the game, Figure 3 plots the payoffs of the second-stage game against the home country's NFA position at each transition point $\tilde{x}_{250}$ for the case of a trade elasticity equal to 4.[23] The payoffs are expressed as country-specific consumption-equivalent variations (τ_1, τ_2).

There are two straightforward takeaways from Figure 3. First, when the home country deviates from cooperation and the foreign country continues to cooperate, the configuration considered in the upper right panel of the figure, the home country is better off deviating, regardless of whether it is a net creditor or a net debtor. Moreover, the home country is better off deviating even when the foreign country retaliates by deviating, as can be evinced by comparing the payoffs for the home country across the bottom two panels of the figure. Accordingly, *deviate* is a dominant strategy for the home country for all transition points. By the same token, *deviate* is also a dominant strategy for the foreign country for all transition points (see the lower left panel of the figure).

Second, since *deviate* is a dominant strategy for both countries in the first stage of the game, the unique Nash equilibrium in the game features both countries opting for their respective nationally oriented welfare function $\mathcal{U}_{j,t}$, for $j = 1, 2$, in the first stage, followed by the open-loop Nash game in the second stage. As countries borrow and lend by trading a diversified portfolio of bonds denominated in both currencies, exchange rate movements do not change the value of their net positions ex post, limiting the scope for cross-country redistribution via monetary measures. With each country responding to the attempt by the other to tilt the terms of trade in favor of its residents, the Nash equilibrium results in inefficient inflation and output stabilization. As shown in the lower right panel in the figure, national welfare falls with the deterioration in the efficiency of the global allocation.

5.3 Efficiency Gains

We now show that much of the benefits from cooperation discussed so far stem from efficiency gains. For this purpose, we compute the cooperative allocations for a range

[23] The country-specific consumption-equivalent variation τ_j, $j = 1, 2$, measures the consumption subsidy/tax that would have to be offered in perpetuity to each household in country j to attain the same level of welfare as under the cooperative policies. See Appendix D.

of welfare weights, allowing us to trace the Pareto frontier, as discussed in Section 4.2.4.

The top panel of Figure 4 shows the Pareto frontier for one of the randomly drawn 1000 transition points. As an example, we consider the case of a trade elasticity of substitution equal to 4. At that transition point, the home country has a net debt balance of 50% of (annualized) output (recall that, for this case, the ergodic probability density function for the NFA position of the home country is shown in the top panel of Figure 1). In the top panel of Figure 4 the X symbol marks the utility levels associated with the non-cooperative outcome. This outcome is inefficient, as indicated by the position of the X symbol well inside the Pareto frontier. In the chart, the broken lines start from the non-cooperative outcome and reach the frontier; they delimit the range of alternative outcomes that would leave either country better off, without making the other country worse off.

The bottom panel of Figure 4 summarizes the Pareto gains (in terms of consumption equivalent variation) for each country for alternative transition points. For comparison, the panel also shows the gains from cooperation assessed using the global welfare function based on symmetric weights.[24] In this case, welfare gains stem from both improved allocation efficiency, and an optimal reallocation of efficiency gains across countries (i.e., gains from redistribution). The fact that, in the figure, the global welfare gains nearly overlaps with the Pareto gains from either country suggests that efficiency is by far the most important driver of our results.

5.4 "Keeping One's House in Order"

In our baseline analysis, we follow the literature by contrasting the case of full cooperation with (open-loop) non-cooperative strategies under commitment. In practice, central banks have the mandate to achieve a small set of nationally oriented objectives, such as domestic price stability and full resource utilization, which can be modeled as flexible inflation targeting, see Section 2.2.3.

[24] To facilitate the comparison across different cases shown, rather than plotting a dot for consumption variation corresponding to each of the 1000 transition points randomly drawn from the ergodic distribution, we fit a fourth-order polynomial function. Apart from the polynomial interpolation, the blue line shown in the figure matches the results also shown in the bottom panel of Figure 1.

In the model, flexible-inflation-targeting policies that place a sufficiently large weight on the output gap can come close to replicating the cooperative case. For instance, with weights $w_\pi = 1$ and $w_y = 10$ for the loss function in Equation (18), the global welfare loss amounts to a modest 3 basis points of consumption. Hence, as external imbalances develop, we expect the incentives to deviate from flexible inflation targeting to be just as strong as for the case of explicit cooperation.

We assess the incentives to deviate from flexible inflation targeting towards objectives that consider the full spectrum of each country's welfare (see Equation 2) using the same two-stage game as in the previous section. In the first stage, each country can choose between *inflation targeting* and *deviate*, given the transition point $\tilde{x}_{250}$. If country j chooses *inflation targeting*, its objective is given by Equation (18); if it chooses *deviate*, its objective is the national welfare function $\mathcal{U}_{j,t}$, for $j = 1, 2$ in Equation (2). In the second stage, the two countries play an open-loop Nash game that determines each country's welfare given the actions selected at the first stage and conditional on the transition point $\tilde{x}_{250}$.

Just like Figure 3, Figure 5 plots the country-specific consumption-equivalent variation of the second stage game against the home country's NFA position for each transition point. The results are strikingly similar to those of Figure 3, but the consumption variation curves are steeper, reflecting the added incentives to move away from sub-optimal, simple objectives towards national welfare functions. Accordingly, inflation targeting is a dominated strategy. In the Nash equilibrium, policymakers in both countries choose *deviate* in the first stage.

5.5 Business Cycle Disturbances

Thus far, for our numerical analysis, we have assumed that technology shocks are the only source of economic disturbances—a hypothesis that we have embraced to enhance comparison with recent literature on the subject of cross-border cooperation. A key implication of modelling the business cycle as exclusively driven by fluctuations in productivity, however, is that cross-border trade in assets tends to be negligible for values of the trade elasticity around unity. Correspondingly, without a sizeable accumulation of net foreign assets in response to shocks, the gains from cooperation are small.

We complete the analysis by allowing for an additional exogenous disturbance, in the form of demand-side, "valuation" shocks (3), as specified in Section 2.1. To assess the gains from cooperation, we set the persistence of the valuation shock process equal to $\rho^{\iota} = 0.95$, and its standard deviation equal to $\sigma^{\iota} = 0.00089$. The unconditional variance of the growth rate of $\iota_{1,t}$ is the same as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016).[25] The valuation shock in the foreign country is parameterized analogously.

The gains from cooperation in a model that includes both valuation and technology shocks are shown in the right-hand panel in the middle row of Figure 9. These gains are much higher than those predicted by our baseline model with technology shocks only (denoted by the shaded area in the graph). Valuation shocks profoundly alter the ergodic distribution of NFA positions. With trade elasticities near unity, the support of the distribution of NFA positions is narrowly concentrated around 0 when focusing exclusively on technology shocks; by contrast, it is broad in the face of valuation shocks. Correspondingly, with both technology and valuation shocks driving business cycles, the gains from cooperation do not fall to 0 for trade elasticities near unity.

6 How General Are Our Results? The currency denomination of bonds and exports

In the rest of the paper, we show that large gains from cooperation persist in leading alternative configurations and extensions of the workhorse monetary model. In the next section (section 7), we will consider alternative structures of the international financial markets. In this section, instead, we focus on the currency denomination of internationally traded assets and goods. We will first allow for an asymmetry in the currency of denomination of debt positions. We will then consider alternative specifications of nominal rigidities in export pricing, to account for, respectively, imperfect

[25] In Albuquerque, Eichenbaum, Luo, and Rebelo (2016) the shock is more persistent, but has a smaller standard deviation.

and/or asymmetric exchange rate pass-through.[26] In all specifications but one, we will modify the baseline considering each of these new assumptions in isolation. In a final case, we will combine assumptions to model the privilege of a country whose currency is "dominant" in both the assets and goods markets.

6.1 International Bonds are Denominated in a Dominant Currency

To study the importance of the currency of denomination of financial instruments, we modify our baseline parameterization by setting $\eta = 0$ in Equation (5). With this change, both countries borrow or lend exclusively in the currency of the home country. Accordingly, policymakers in this country gain a clear advantage. Recall that in our baseline (with symmetric bond portfolios), the monetary policy instruments of the two countries are equally effective at influencing the real value of the net foreign asset position. With the change in assumptions of this section, only the home policymakers can affect the real value of their country's nominal foreign liabilities or assets by affecting domestic prices. This prerogative magnifies the temptation for the home policymakers to act strategically and thus boosts the gains from cooperation relative to the symmetric bond case discussed so far. The higher gains from cooperation with asymmetric portfolios are apparent from comparing Figure 6 and Figure 2, both showing the gains from cooperation for different values of the trade elasticity under the same setup described in Section 5.1, but for the currency denomination of bonds.

The asymmetry in the incentives to deviate are illustrated by Figure 7, which, based on the same two-stage game described in Section 5.2, shows the payoffs for each country depending on the transition point. This figure highlights three key points. First, relative to the symmetric bond case in Figure 3, when bonds are denominated in the home country's currency, the welfare incentive to play strategically is significantly stronger for the home country, significantly weaker for the foreign one. Second, the (now larger) gains the home country can seek by deviating are smaller than the loss it imposes on the foreign country regardless of the response of the foreign country.

[26] See Section A.8.1 of the appendix for details on the model setup.

Third and final, when both countries choose to deviate from cooperation, the home policymakers clearly have the upper hand in the game—the case in the bottom right panel of the figure.[27]

As in our baseline, the size of the NFA position continues to drive the gains from cooperation. With debt denominated in the home country's currency, however, redistribution plays a non-negligible role in driving the welfare of the two countries apart. The top panel of Figure 8 focuses again on a transition point in which the debt of the home country to the foreign country amounts to 50% of the output of the home country. By engineering a devaluation of the stock of nominal debt, the non-cooperative policy moves the allocation not only well inside the Pareto frontier, but also far away from the equal-weight cooperative point. The strong redistribution is obviously in favor of the home country.

The welfare gains and the Pareto efficiency gains from cooperation are plotted in the bottom panel of the figure for each country. While, relative to our baseline, redistribution now plays a bigger role in our results, this panel confirms that efficiency gains continue to be sizable for both countries (albeit lower for the home country than for the foreign country). As each country is tempted to pursue redistributive strategies through a contraction that improves the terms of trade, ultimately those strategies result in an inefficient drop in economic activity.

6.2 Exports are Priced in Either the Local or a Dominant Currency (Incomplete Pass-through)

The baseline model features producer-currency-pricing. Accordingly, exchange rate fluctuations are reflected in import prices in local currency one for one in both countries; exchange rate pass-through is symmetrically *full*. We now show that the gains from cooperation are, on average, larger and more disperse when exchange rate pass-through is either incomplete or asymmetric across the two countries. In this new set

[27] These results could be interpreted as one dimension of the privilege enjoyed by countries that can borrow and lend with bonds denominated in their own currency. In this respect, a note of caution is in order. Extrapolating from the results in the text, if a country decided to take advantage of such privilege adopting nationally oriented policies, dynamically, the basis for that privilege could be expected to come under stress quickly—the cost of borrowing in its own currency could be expected to rise rather steeply.

of exercises, we first revert to assuming a symmetric portfolio of international bonds; we then conduct a specific exercise assuming that the currency of the home country is dominant in the international markets for both assets and goods.

For a symmetric bond portfolio, the top row in Figure 9 shows the gains from cooperation for different values of the trade elasticity of substitution, when exports are either denominated in the currency of the market of destination—local-currency pricing or LCP— or denominated in the home country's currency—dominant-currency-pricing or DCP. The gray-shaded area shows the range of outcomes under the baseline model. Exchange rate pass-through impinges on the policy trade-offs faced by policymakers. As discussed early on by the literature (see, e.g, Devereux and Engel 2003), incomplete pass-through tends to magnify the domestic effects and cross-border spillover of monetary policy on employment. Accordingly, incomplete pass-through strengthens the incentive for policymakers to pursue nationally-oriented policies aimed at saving on labor effort.[28]

Comparing the LCP and DCP panels in Figure 9, it is apparent that the gains from cooperation are higher under local-currency-pricing, when pass-through of exchange rate movements to trade prices is incomplete across all border, than under dominant-currency-pricing, when the pass-through is less than full only for the imports by the (home) country issuing the dominant currency. These results suggest that, acting strategically, monetary policymakers become more aggressive, the lower the degree of overall pass-through.

Everything else equal, however, the gains from cooperation are highest when the prices of both bonds and goods traded across borders are denominated exclusively in a dominant currency—the case depicted by the left panel of the middle row of Figure 9. A key takeaway is that, in a world where one currency is dominant in both financial and real markets, imbalances strengthen the temptation of the dominant

[28] Devereux and Engel (2003) studies how local-currency-pricing affects cooperative and non-cooperative policies. In a model with complete markets, they focus on the special case in which the trade elasticity of substitution and the intertemporal elasticity of substitution are the inverse of each other, see Cole and Obstfeld (1991), which lends analytical tractability but masks the differences between cooperative and non-cooperative equilibria. Relatedly, Fujiwara and Wang (2017) show that the welfare gains from cooperation are larger under local-currency-pricing than producer-currency-pricing in a model similar to ours but with complete markets. However, given their choices of parameters (including flexible wages) and initial conditions, the welfare gains from cooperation remain negligible regardless of the assumptions about the currency of invoicing.

country to break cooperative arrangements, much more than for other countries.[29]

7 Financial Markets and Risk Sharing

In our baseline, the accumulation of non-contingent foreign debt plays a key role in shaping the trade-offs faced by strategic policymakers, hence their incentives to deviate from cooperative practices. Indeed, a remarkable result from our quantitative analysis so far is that, as long as debt is close to zero, the gains from cooperation remain negligible in all the extensions of our baseline model. In this respect, not only does our quantitative analysis validate the main message from the analytical Section 3, stressing the role of external imbalances in driving the incentive to act strategically, but it also resonates with results in Obstfeld and Rogoff (2002), Devereux and Engel (2003), and Corsetti and Pesenti (2005). These related analyses found that, with zero debt *and* a unit trade elasticity, a case in which the economic allocations are independent of financial markets arrangements in the face of technology shocks, the gains from cooperation are negligible or altogether absent.

In this section, we extend our study to economies operating under either complete markets or financial autarky, where non-contingent debt is no longer a state variable.[30] We will show that, first, the gains from cooperation are still contingent on economic conditions, with real wages replacing debt in providing an efficient proxy for these conditions. Second, and most crucially, the magnitude of these gains still depend on cross-border *contingent* financial flows (or lack thereof). For ease of comparison with the literature and our baseline, throughout the section, we will restrict our attention to economies with only technology shocks, focusing on the case of symmetrically complete pass-through (PCP).

[29] This in part reflects the redistributive dimensions of strategic policies discussed in the previous subsection—and resonates with well-known results in the literature, stressing that the dominant currency country may have little incentive to pursue cooperative arrangements in monetary policy (see Corsetti and Pesenti 2005).

[30] See Section A.8.3 of the appendix for details of the model setup.

7.1 Fragility of Cooperation under Complete Markets

In our incomplete markets baseline with international financial flows limited to non-state-contingent bonds, the incentives to deviate from cooperation grow with the outstanding stock of net foreign assets of a country. Under complete markets, financial contracts may still give rise to large state-contingent flows of resources across borders. But these cross-border financial obligations play a very different role, relative to debt, in shaping the policymakers trade-offs that give rise to the temptation to act strategically. The reason is as follows.

To start, we note that, when a country experiences a sequence of positive shocks, their cumulative effects push up the real wage, and residents in the country enjoy higher consumption (the marginal utility of consumption is low). At that point, national policymakers perceive higher benefits from pursuing nominal and real exchange rate appreciation, to trade off, at the margin, higher leisure with some reduction in consumption. Real wages thus emerge as the key proxy to index the gains from cooperation.

Nonetheless, it is the cross-border insurance provided by financial contracts that defines the intensity of the incentive to act strategically faced by strategic policymakers. To appreciate this point, focus on the bottom left panel of Figure 9, which plots the gains from cooperation under complete markets for different values of the trade elasticity. This figure shows that the gains from cooperation (correspondingly, the spillover effects of country-specific monetary policy) are monotonically increasing, and become economically significant for a trade elasticity sufficiently above unity. As discussed in previous sections, for elasticities around 1, production risk is *de facto* insured by terms of trade movements (see Cole and Obstfeld 1991), and trade in assets is irrelevant—implying that even under complete markets there are no financial flows across the two countries. However, for higher values of the trade elasticity, terms of trade movements become progressively less relevant. Correspondingly, state contingent financial flows play an increasingly important role in insuring against output fluctuations. These state-contingent flows buffer the social costs of a unilateral monetary contraction (aiming to improve their country's terms of trade), as they accrue to residents in the country contingent on the implied fall in output. Since, in addition, at higher trade elasticities residents can easily substitute domestic goods with cheaper imports, strategic policymakers will perceive an increasingly favorable trade-off between consumption and labor efforts—especially tempting when the real wage is high, a configuration that points to low marginal utility of consumption.

We close our reasoning by observing that, since we allow for both nominal price and wage rigidities, monetary policymakers face a meaningful inflation-output trade-off independently of the terms-of-trade monopoly distortion.[31] Because of this trade-off, shocks can move the economy away from the efficient allocation in a persistent manner, and the real wage conveys information above and beyond the marginal product of labor: it generally includes a wedge over the marginal product that depends on how policymakers have responded to (productivity) disturbances in the past. This wedge expands the range of the gains from cooperation that our model predicts for any given choice of the trade elasticity—reflected, in our Figure 9, in the vertical distance between the 5^{th} and the 95^{th} percentiles of the realized distribution of the gains for the different transition points (denoted by the dotted lines). It also sharpens the link between the real wage and the gains from cooperation. Indeed, regressing the gains from cooperation on domestic and foreign real wages (and their squares), we attain an R^2 statistic that varies between 0.94 and 1.00 depending on the value of the trade elasticity (in the range from 0.65 to 4).

But, while real wage distortions weigh on the incentive to act strategically, the gains from cooperation under complete market are mostly driven by the correction of the terms-of-trade externality. With financial contracts insuring private consumption against output and income fluctuations caused by self-oriented monetary contractions, compared to our baseline, the temptation to manipulate the terms of trade is less sensitive to economic conditions, i.e., the evolution of productivity. As shown by our figure, at relatively high level of trade elasticities, above two, the gains from cooperation are significantly above zero not only at the high end, but also at the lower end of the distribution in the figure (corresponding to lower realizations of the

[31] The so-called "divine coincidence" fails in our model. Intuitively, the number of instruments that monetary policymakers control is insufficient to stabilize both price and wage inflation in the two countries.

real wages). This result is in sharp contrast to our baseline case with only non-state-contingent bonds (the shaded area in the figure), where, reflecting the high dispersion in the accumulation of foreign debt, the 5^{th} percentile of the gains touches zero.

7.2 Financial Autarky Disciplines Strategic Behavior

Under financial autarky, real wages can be expected to play a role in driving the gains from cooperation similar to their role under complete markets. We confirm this insight for economies that do not trade assets internationally by regressing these gains on the real wage at home and abroad (and their squares) for each transition point. The R^2 statistic is 1.00 irrespective of the trade elasticity (in the range between 0.65 and 4 that we consider for this elasticity).

However, without trade in assets, the gains from cooperation are dramatically diminished, as shown in the bottom right panel of Figure 9. Unlike under complete markets, absent risk sharing via financial markets, agents are no longer able to insure their consumption from undesired effects of non-cooperative policies. At high real wages, the perceived gains from attempting to improve leisure by improving a country's terms of trade are moderated by the high costs in terms of consumption.[32]

8 Conclusion

Our results strengthen the theoretical support for the view that the gains of full cooperation across countries for monetary policy may be small relative to best practice flexible inflation targeting. Nonetheless, our analysis also shows that the range of theoretical results from open economy macro theory is broader than previously acknowledged, and that policies consistent with a non-cooperative Nash equilibrium can be consequential for global welfare. Our contributions to the literature can be summarized in four points.

First and foremost, using the same model that has lent support to the claim

[32] One may observe that, in Figure 9, gains from cooperation that are non-negligible (yet quite small) occur only for low trade elasticities, below 0.7. The reason is that at these low elasticities the volatility of the exchange rate in response to fundamental shocks and policies is quite high. Unilateral deviations from cooperation result in strong relative price movements hence in non-negligible cross-border spillovers.

by Obstfeld and Rogoff (2002), we have shown that there are empirically plausible conditions that make the gains from cooperation several times larger than the cost of economic fluctuations. Our second point is methodological: a full appreciation of the welfare gains from cooperation requires an assessment conditional on economic conditions, as they evolve endogenously along possible histories of the economy. Our third point concerns the nature of these gains. While non-cooperative strategies that attempt to improve the terms of trade of a country are redistributive in their aim, they inefficiently reduce economic activity. Accordingly, we find that cooperation leads to large Pareto efficiency gains.

Our fourth and last point stresses that financial frictions are not a necessary precondition for monetary policy to have large cross-border spillovers. If anything, our results suggest that the temptation to deviate from cooperation can be expected to be stronger in a financially globalized world, where residents in a country can borrow/lend abroad accumulating large imbalances (our baseline), or trade assets with state-contingent cash flows to share risk (our complete market specification), relative to a world with no trade in cross-border financial markets (our financial autarky specification). We have shown that, with borrowing and lending, the gains from cooperation grow quadratically with the net-foreign-asset position. With complete markets, the gains are modulated by wages in the two countries, but financial flows still play a fundamental role. Namely, the gains from cooperation are larger the greater is the reliance of private agents on international financial markets to share risk.

During the last decades, the world has witnessed substantial and persistent accumulation of external debt, accompanied by remarkable changes in relative incomes and wages. Especially in the aftermath of the Global Financial Crisis, and more recently in response to the COVID-19 pandemic, macroeconomic stabilization has been facing increasingly complex challenges. With persistent external and internal imbalances, domestic policymakers may become less tolerant of the requirements of good behavior from a global perspective. Holding foreign policies constant, the perceived trade-offs may tilt in favor of nationally oriented policies, which, breaking away from the post-Bretton Woods equilibrium, may be pursued in an antagonistic way. The risk is that strong policy actions may end up magnifying external spillovers, especially if they trigger a spiral of retaliatory actions.

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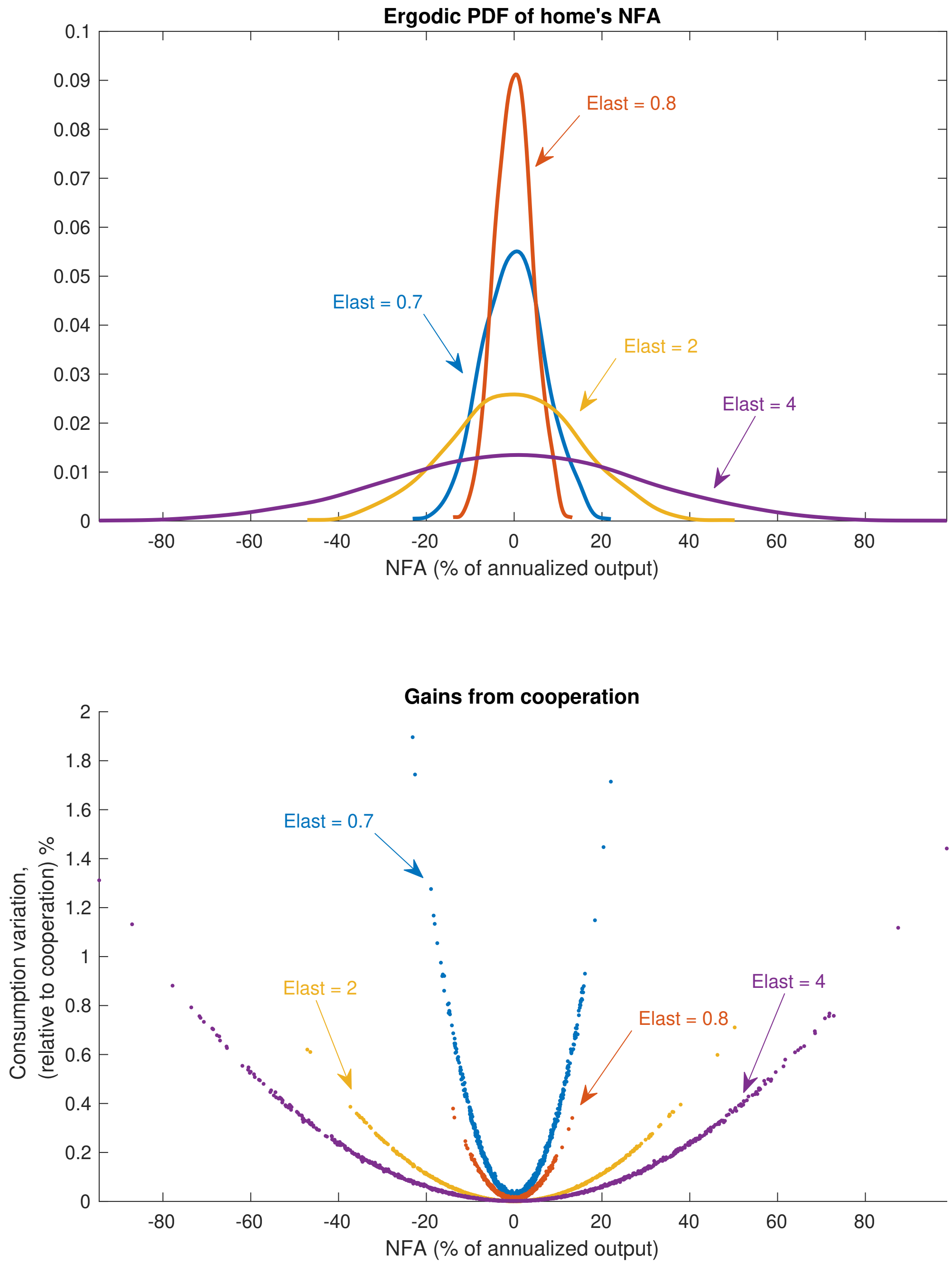
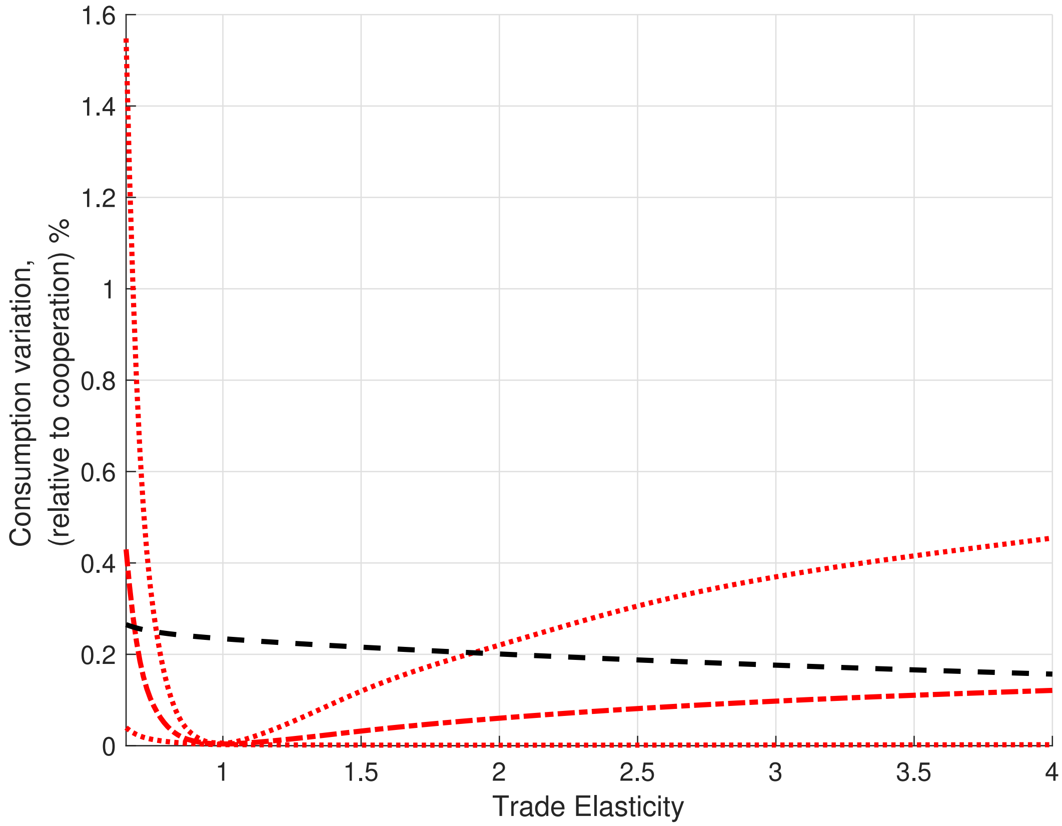
Figure 1: The Distribution of the Net Foreign Assets and Gains from Cooperation for Alternative Values of the Elasticity of Substitution between Traded Goods



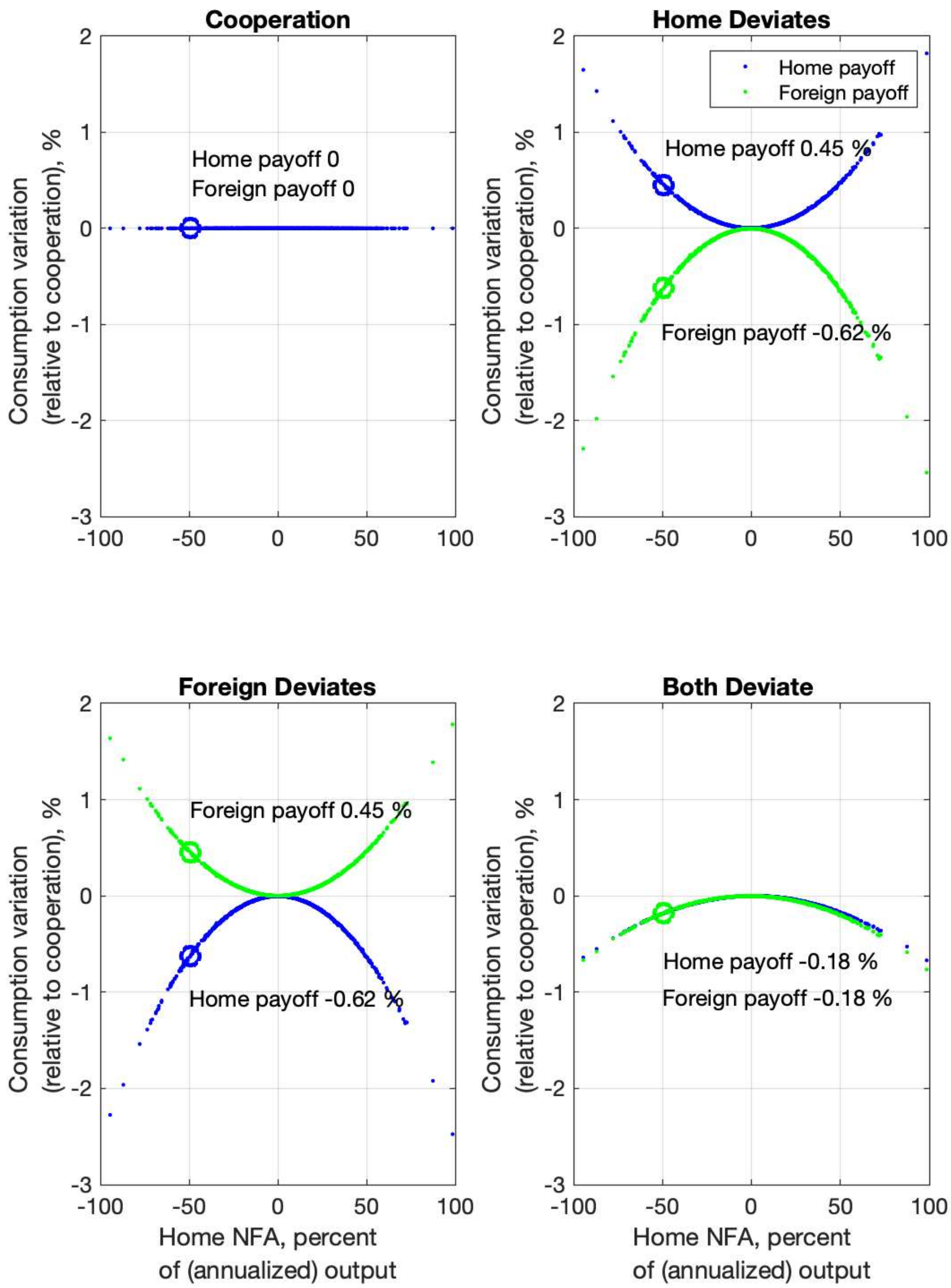
Note: In the figure, "NFA" stands for net foreign assets. The top panel shows the ergodic probability density function of the NFA position for the home country for alternative values of the trade elasticity of substitution. For the same trade elasticities, the bottom panel shows the expected gain from continuing to cooperate relative to adopting nationally oriented policies. The gains are evaluated at 1000 points randomly drawn from the ergodic distribution under cooperative policies. The welfare difference between the two policy arrangements is translated into a consumption equivalent variation as described in Section 4.2.1.

Figure 2: The Gains from Cooperation as a Function of the Elasticity of Substitution between Traded Goods—Symmetric Portfolio



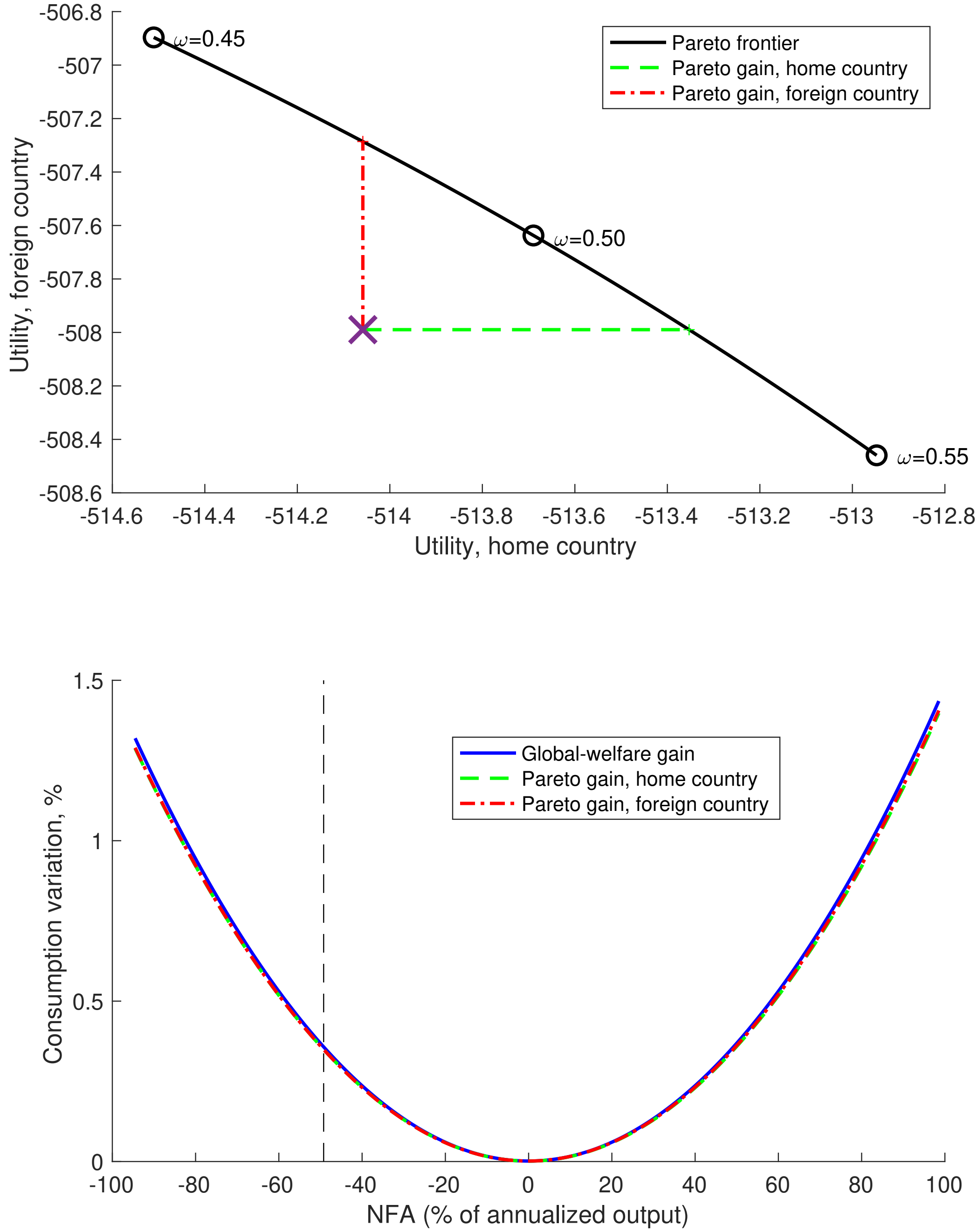
Note: The figure shows the mean gain from continuing to cooperate relative to adopting nationally oriented policies based on 1000 points randomly drawn from the ergodic distribution under cooperative policies (the dashed-dotted line). The welfare difference between the two policy arrangements is translated into a consumption equivalent variation as described in Section 4.2.1. The 5th and 95th percentiles (the dotted lines) refer to the realized distribution of gains for the different transition points. For comparison, the figure also shows the cost of business cycles (the dashed line), computed as described in Section 4.2.2.

Figure 3: Two-Stage Game—Symmetric Portfolio and Trade Elasticity = 4



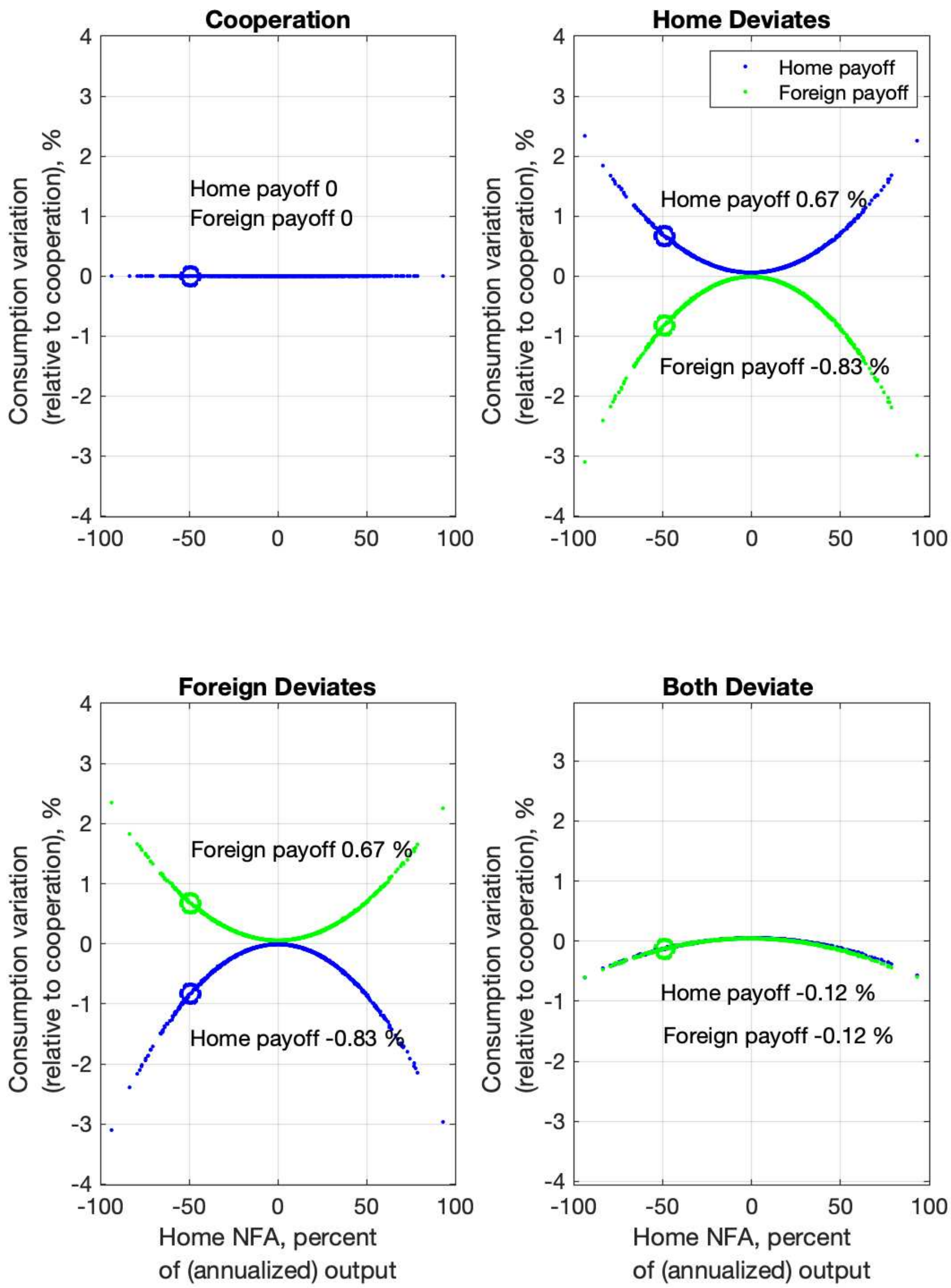
Note: The abbreviation "NFA" stands for net foreign assets. To account for the incentives to deviate from the cooperative behavior, we rely on the two-stage game described in Section 4.2.3. At each transition point, in the first stage, we let each country choose between *cooperate* or *deviate*; in the second stage, we let the countries play an open-loop Nash game conditional on their preference choices in the first stage. For all combinations of actions in the first stage of the game, the figure plots the payoff of the second-stage game against the home country's NFA position at each transition point. The payoffs are expressed as country-specific consumption-equivalent variations (τ_1, τ_2). By construction, τ_1 and τ_2 are 0 regardless of the NFA position if both countries choose *cooperate*.

Figure 4: Pareto Frontier and Efficiency Gains—Symmetric Portfolio and Trade Elasticity = 4



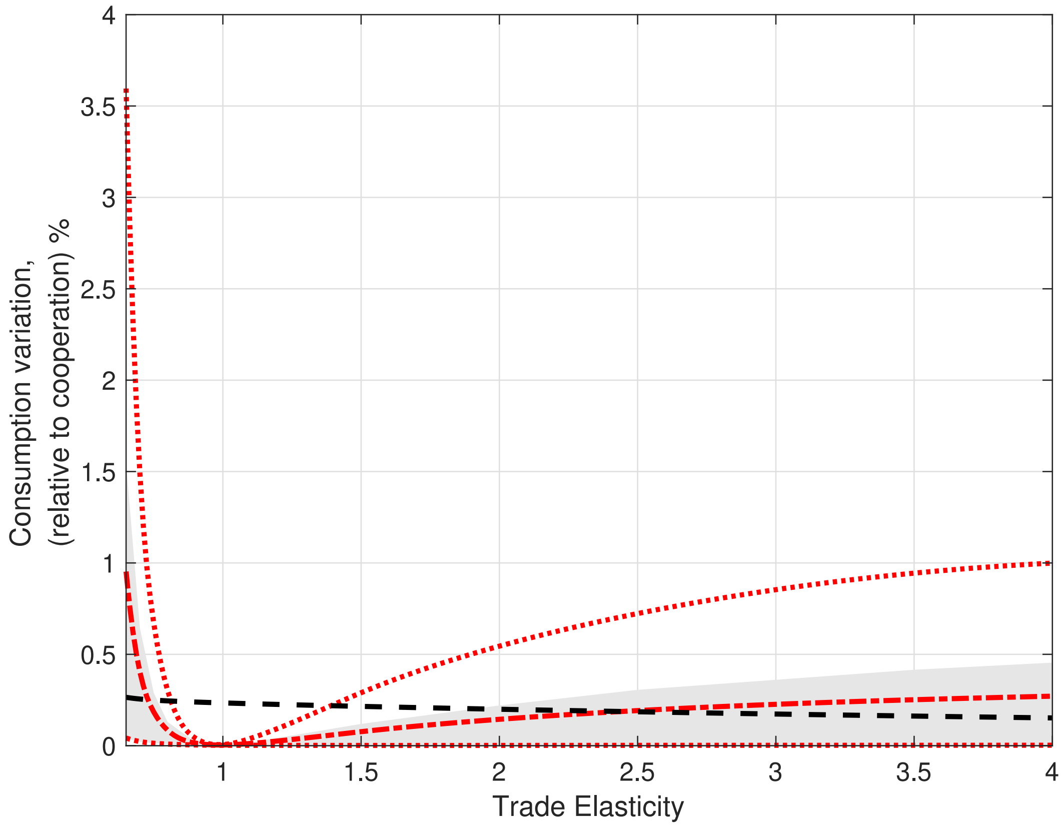
Note: The top panel shows the Pareto frontier (the solid black line) for one of the randomly drawn 1000 transition points. At that transition point, the home country has a net debt balance of 50% of (annualized) output. The X symbol in the top panel marks the utility associated with the non-cooperative allocation. The bottom panel summarizes the Pareto gains for each country for alternative transition points. The abbreviation "NFA" stands for net foreign assets. The vertical dashed line denotes the NFA position for the transition point used for the Pareto frontier shown in the top panel. The bottom panel also shows the gains from cooperation from the global welfare function based on symmetric weights.

Figure 5: Two-Stage Game, Inflation Targeting—Symmetric Portfolio and Trade Elasticity = 4



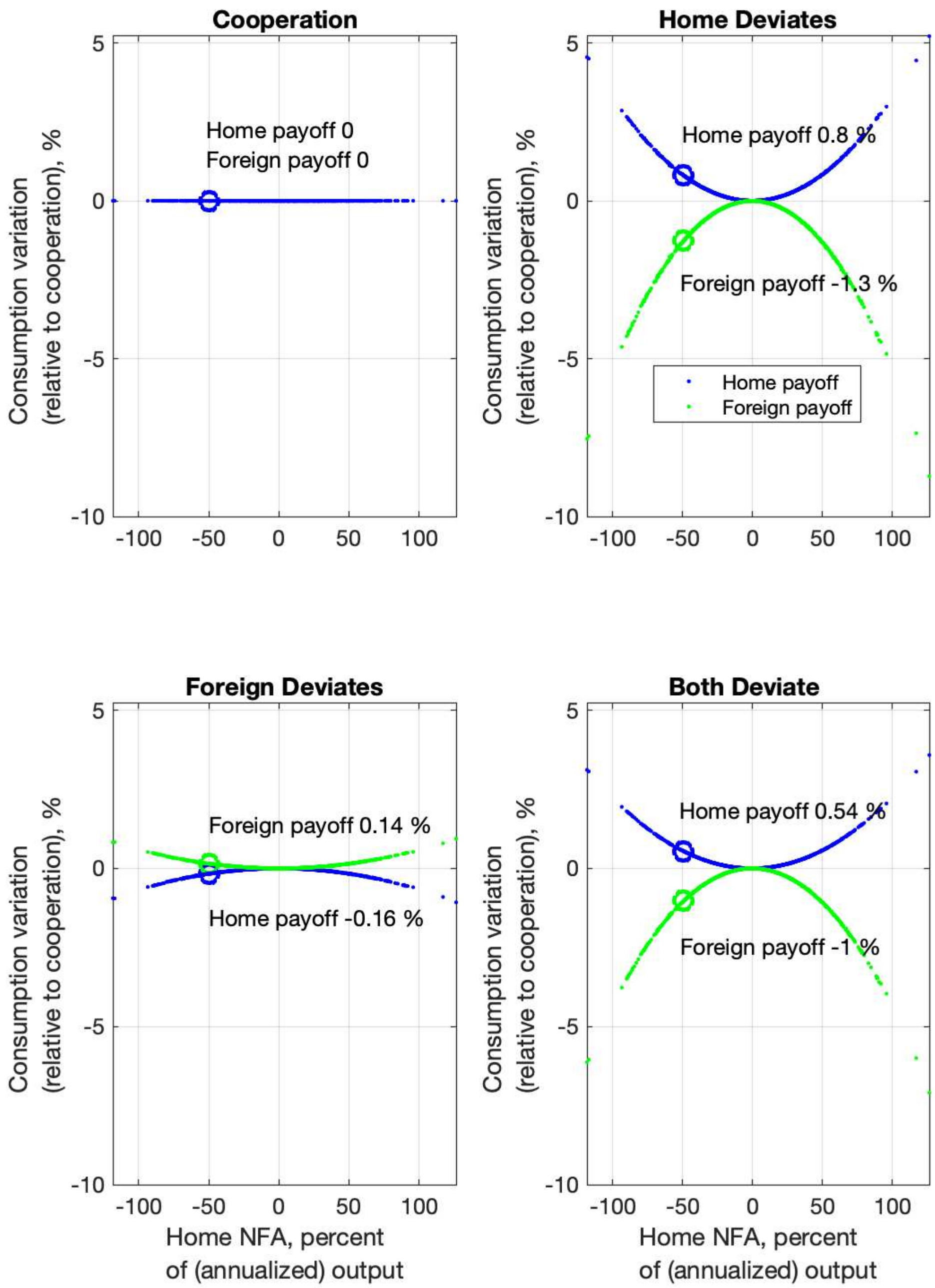
Note: The abbreviation "NFA" stands for net foreign assets. To account for the incentives to deviate from the cooperative behavior, we rely on the two-stage game described in Section 4.2.3. At each transition point, in the first stage, we let each country choose between *inflation targeting* or *deviate towards national policies*; in the second stage, we let the countries play an open-loop Nash game conditional on their preference choices in the first stage. For all combinations of actions in the first stage of the game, the figure plots the payoff of the second-stage game against the home country's NFA position at each transition point. The payoffs are expressed as country-specific consumption-equivalent variations (τ_1, τ_2). By construction, τ_1 and τ_2 are 0 regardless of the net- foreign-asset position if both countries choose *cooperate*.

Figure 6: The Gains from Cooperation as a Function of the Elasticity of Substitution between Traded goods—Asymmetric Portfolio



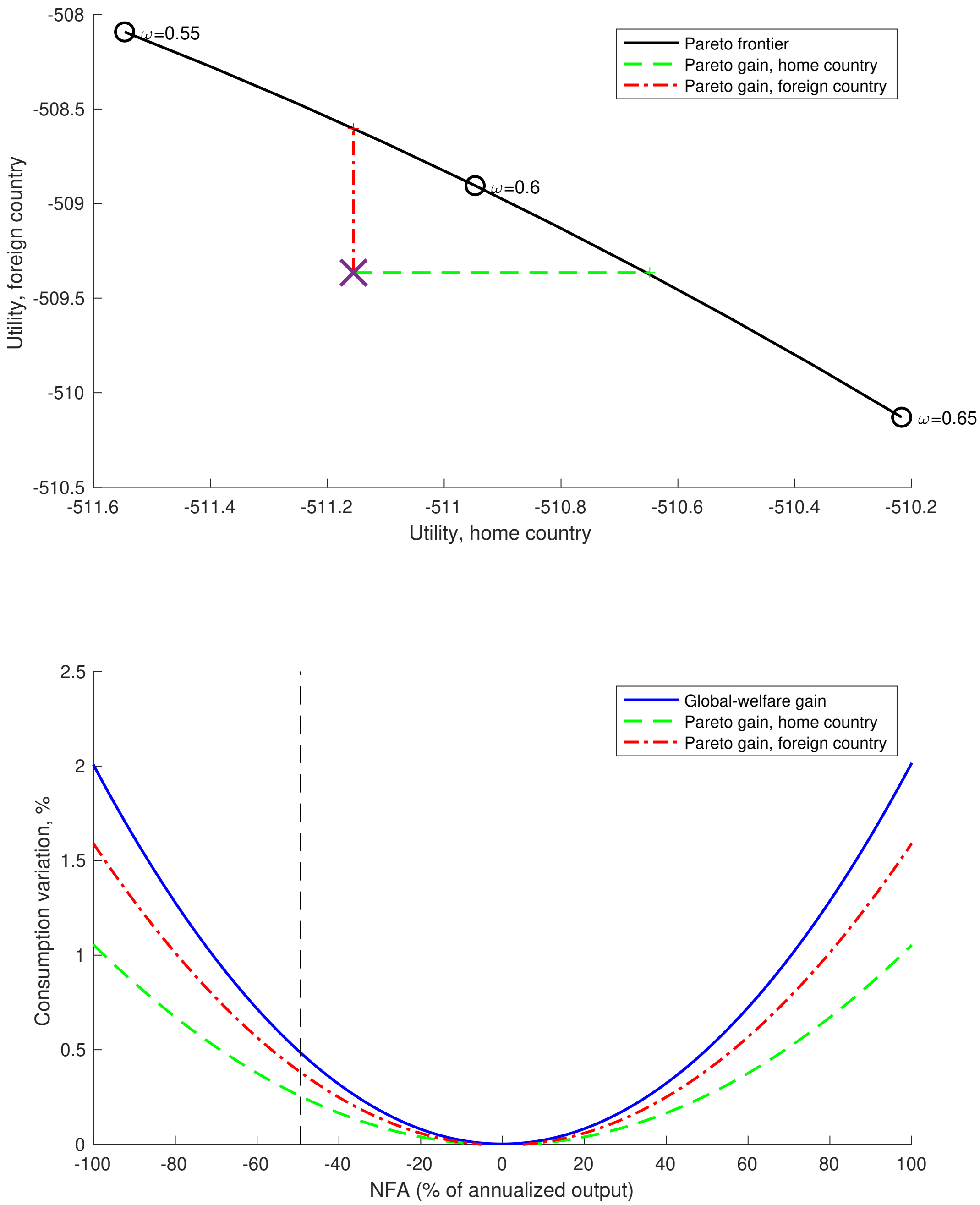
Note: The figure shows the mean gain from continuing to cooperate relative to adopting nationally oriented policies based on 1000 points randomly drawn from the ergodic distribution under cooperative policies (the dashed-dotted line). The welfare difference between the two policy arrangements is then translated into a consumption equivalent variation as described in Section 4.2.1. The 5th and 95th percentiles (the dotted lines) refer to the realized distribution of gains for the different transition points. For comparison, the figure also shows the cost of business cycles (the dashed line), computed as described in Section 4.2.2.

Figure 7: Two-Stage Game— Asymmetric Portfolio and Trade Elasticity = 4



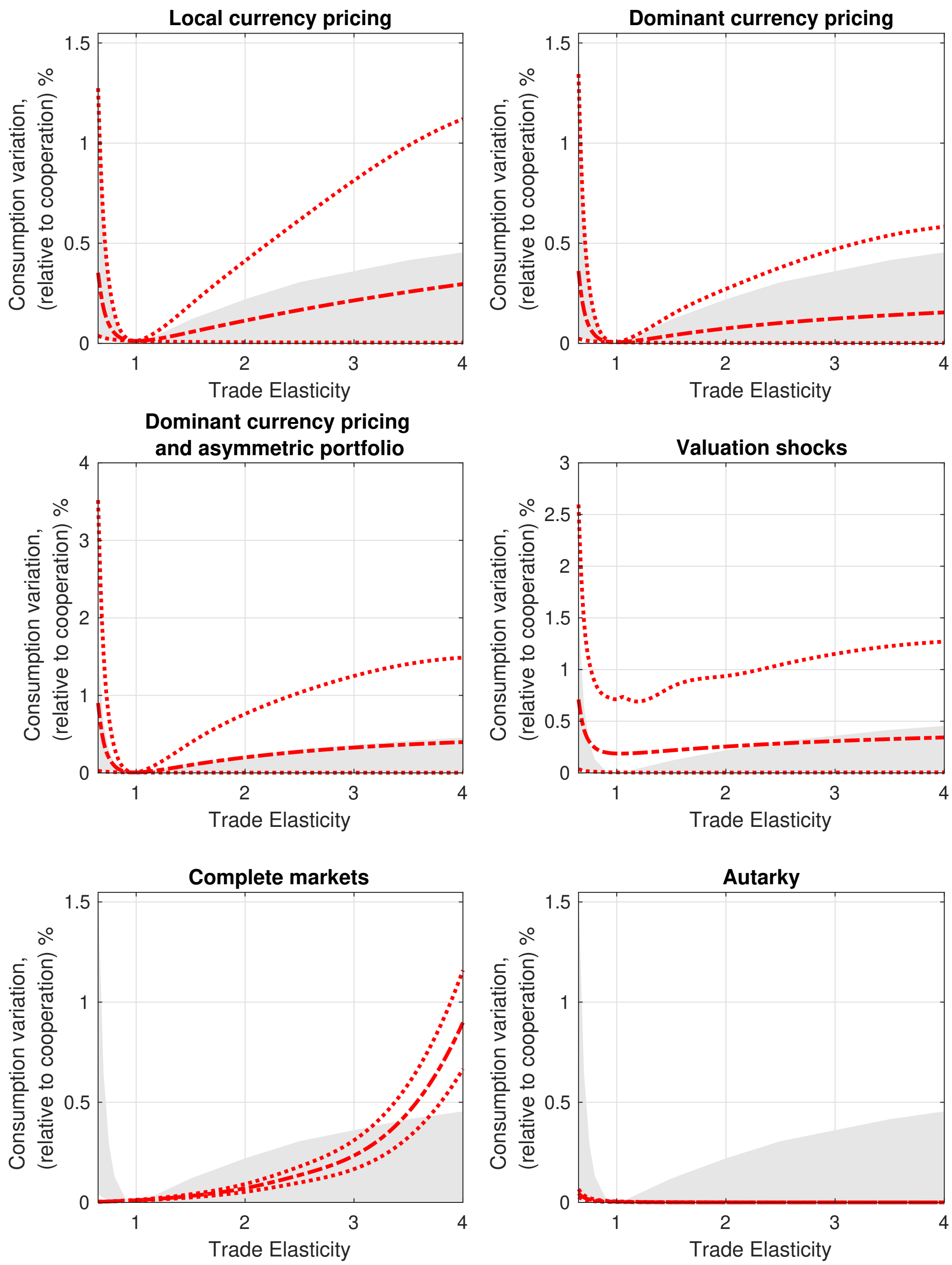
Note: The abbreviation "NFA" stands for net foreign assets. To account for the incentives to deviate from the cooperative behavior, we rely on the two-stage game described in Section 4.2.3. At each transition point, in the first stage we let each country choose between *cooperate* or *deviate*; in the second stage, we let the countries play an open-loop Nash game conditional on their preference choices in the first stage. For all combinations of actions in the first stage of the game, the figure plots the payoff of the second-stage game against the home country's NFA position at each transition point. The payoffs are expressed as country-specific consumption-equivalent variations (τ_1, τ_2). By construction, τ_1 and τ_2 are 0 regardless of the NFA position if both countries choose *cooperate*.

Figure 8: Pareto Frontier and Efficiency Gains with Asymmetric Portfolio and Trade Elasticity = 4



Note: The top panel shows the Pareto frontier (the solid black line) for one of the randomly drawn 1000 transition points. At that transition point, the home country has a net debt balance of 50% of (annualized) output. The X symbol in the top panel marks the utility associated with the non-cooperative allocation. The bottom panel summarizes the Pareto gains for each country for alternative transition points. The abbreviation "NFA" stands for net foreign assets. The vertical dashed line denotes the NFA position for the transition point used for the Pareto frontier shown in the top panel. The bottom panel also shows the gains from cooperation from the global welfare function based on symmetric weights.

Figure 9: The Importance of Exchange Rate Pass-through, Shock Sources, and Financial Arrangements for the Gains from Cooperation



Note: The figure shows the mean gain from continuing to cooperate relative to adopting nationally oriented policies based on 1000 points randomly drawn from the ergodic distribution under cooperative policies (the dashed-dotted line). The welfare difference between the two policy arrangements is then translated into a consumption equivalent variation as described in Section 4.2.1. The 5th and 95th percentiles (the dotted lines) refer to the realized distribution of gains for the different transition points. Each panel focuses on an alternative model as discussed in the text. For comparison, the shaded area in each panel shows the 5th-95th interval for the gains from cooperation for the baseline model with incomplete markets and a symmetric portfolio of international non-state-contingent bonds.

A Appendix: Model Description

The description of the model in this appendix complements the description in the main text. The focus here is on derivation of the private sector's equilibrium conditions. This section of the appendix also includes a description of model variants: local- and dominant-currency pricing, complete financial markets, and financial autarky. The last section of this appendix returns to the baseline model with non-state-contingent bonds only for international financial flows, incomplete markets, and producer-currency-pricing to express household utility as an indirect function of relative prices, wage and price dispersion measures, and the net foreign asset position.

A.1 Relative Prices

We start with some definitions of relative prices that will recur through the appendix and that will allow us to write the private sector's equilibrium conditions more compactly. Define

$$\nu_{j,t} = \frac{P^m_{j,t}}{P^d_{j,t}} \tag{A.1}$$

for $j = [1, 2]$ to be the relative price of the imported good in local currency $P^m_{j,t}$ and the exported good in local currency $P^d_{j,t}$ of country j. We denote by

$$\delta_{1,t} = \frac{P^m_{1,t}}{e_{1,t} P^m_{2,t}} = \frac{\nu_{1,t} P^d_{1,t}}{\nu_{2,t} e_{1,t} P^d_{2,t}}. \tag{A.2}$$

the terms of trade for country 1, the ratio of the price of imports to the price of exports expressed in a the currency of country 1. The term $e_{1,t}$ is the nominal exchange rate.

It will also be useful to keep in mind that, using the first-order conditions for the problem of minimizing the cost of producing the final consumption good (spelled out in Equation A.10 in the next section), the price of the final consumption good relative to the locally produced good in each country, $\frac{P^c_{1,t}}{P^d_{1,t}}$ and $\frac{P^c_{2,t}}{P^d_{2,t}}$, can be expressed as

$$\frac{P^c_{1,t}}{P^d_{1,t}} = \left[\omega^c_1 + (1-\omega^c_1)\nu_{1,t}^{-\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{1,t}^{-\rho^c}, \tag{A.3}$$

$$\frac{P^c_{2,t}}{P^d_{2,t}} = \left[\omega^c_2 + (1-\omega^c_2)\nu_{2,t}^{-\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c}. \tag{A.4}$$

Notice that the term $F_{j,t}$ for $j \in [1, 2]$ is defined (implicitly) by these two last equations.

A.2 Households

Relative to the main text, we abstract from valuation shocks for now and introduce them in Section A.8. Households solve the maximization problem

$$\max_{\substack{C_{1,t+j}, B_{11,t+j}, \\ B_{12,t+j}, D_{1,t+1+j|t+j}}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{1,t+j} - \kappa C_{1,t+j-1} \right) - \frac{\chi_0}{1+\chi} L_{1,t+j}^{1+\chi} \right\}$$

subject to

$$P^c_{1,t+j} C_{1,t+j} + \frac{P^b_{1,t+j} B_{11,t+j} + e_{1,t+j} P^b_{2,t+j} B_{12,t+j}}{\phi^b_{1,t+j}} + \int_S P^D_{1,t+1+j|t+j} D_{1,t+1+j|t+j} =$$
$$W_{1,t+j} L_{1,t+j} + B_{11,t-1+j} + e_{1,t+j} B_{12,t-1+j} + D_{1,t+j|t-1+j} + \Psi_{1,t+j}, \text{ and to}$$
$$\eta B_{11,t+j} = (1-\eta) e_{1,t+j} B_{12,t+j}.$$

The first-order conditions associated with this problem can be written as

$$MU_{1,t} = \left(\frac{1}{1 - \kappa \frac{C_{1,t-1}}{C_{1,t}}} - \frac{\beta\kappa}{\frac{C_{1,t+1}}{C_{1,t}} - \kappa} \right) \frac{1}{C_{1,t}}, \tag{A.5}$$

$$\frac{1}{1+R_{1,t}} = \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{P^d_{1,t}}{P^d_{1,t+1}} \left(\frac{F_{1,t+1}}{F_{1,t}} \right)^{\rho^c} \right\}, \tag{A.6}$$

$$\tilde{P}^b_{1,t} \equiv (1-\eta) P^b_{1,t} + \eta P^b_{2,t},$$
$$= \phi^b_{1,t} \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\delta_{1,t+1}}{\delta_{1,t}} \frac{\nu_{1,t+1}}{\nu_{1,t}} \frac{\nu_{2,t}}{\nu_{2,t+1}} \left(\frac{F_{1,t+1}}{F_{1,t}} \right)^{\rho^c} \frac{P^d_{2,t}}{P^d_{2,t+1}} \left((1-\eta) \frac{e_{1,t}}{e_{1,t+1}} + \eta \right) \right\}. \tag{A.7}$$

The term $MU_{1,t}$ denotes the marginal utility of consumption. The intermediation cost $\phi^b_{1,t}$ is given by

$$\phi^b_{1,t} = \exp \left(\phi^b \frac{B^A_{11,t} + e_{1,t} B^A_{12,t}}{P^d_{1,t} C^{d,A}_{1,t} + e_{1,t} P^m_{2,t} M^A_{2,t}} \right). \tag{A.8}$$

This cost depends on the aggregate bond holdings at the country level and aggregate output. Thus an individual household does not take into account the effects of its choices on the intermediation costs. In equilibrium, it is of course that case that $B^A_{11,t} = B_{11,t}$, $B^A_{12,t} = B_{12,t}$, $C^{d,A}_{1,t} = C^d_{1,t}$, and $M^A_{2,t} = M_{2,t}$.

Our analysis tracks the NFA position of the two countries. As we solve the model to second-order accuracy, restricting the set of international assets to a single bond, with the bond denominated in the currency of one country, introduces an asymmetry between the two countries, even if the two countries mirror each other in every other respect. With two bonds that are denominated in the two countries' respective currencies we can eliminate this asymmetry by requiring the two countries to hold the bonds in equal proportion, i.e., $\eta = 0.5$.

Note that we have not provided a first-order condition for the household's labor. In the next section we discuss how sticky nominal wages are determined. As is customary, we define a "desired nominal wage," $\tilde{W}_{1,t}$ that, once deflated by the domestic price level, equals the marginal rate of substitution between the disutility of labor and consumption

$$\frac{\tilde{W}_{1,t}}{P^d_{1,t}} = \chi_{0,1} \frac{L^{\chi_1}_{1,t}}{MU_{1,t}} \frac{1}{F^{\rho^c}_{1,t}}. \tag{A.9}$$

The final consumption good combines the home good (priced at $P^d_{1,t}$) and the foreign good (priced at $P^m_{1,t}$) according to

$$C_{1,t} = \left((\omega^c_1)^{\frac{\rho^c}{1+\rho^c}} \left(C^d_{1,t}\right)^{\frac{1}{1+\rho^c}} + (\omega^m_1)^{\frac{\rho^c}{1+\rho^c}} \left(M_{1,t}\right)^{\frac{1}{1+\rho^c}} \right)^{1+\rho^c}, \tag{A.10}$$

with the first-order conditions of cost minimization given by

$$C^d_{1,t} = \omega^c_1 \left(\frac{P^c_{1,t}}{P^d_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t} = \omega^c_1 F^{-(1+\rho^c)}_{1,t} C_{1,t}, \tag{A.11}$$

$$M_{1,t} = (1-\omega^c_1) \left(\frac{P^c_{1,t}}{P^m_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t} = (1-\omega^c_1) F^{-(1+\rho^c)}_{1,t} \nu^{-\frac{1+\rho^c}{\rho^c}}_{1,t} C_{1,t}. \tag{A.12}$$

Note that we used $\omega^m_1 = 1 - \omega^c_1$.

A.3 Sticky Nominal Wages

Unions introduce distinguishing characteristics to the homogenous labor $L_{1,t}$ supplied by the households and resell these services to bundlers. The bundlers sell the aggregate labor services $L^d_{1,t}$ to the producers of good. It is

$$L^d_{1,t} = \left[\int_0^1 L_{1,t}(h)^{\frac{1}{1+\theta^w}} dh \right]^{1+\theta^w}. \tag{A.13}$$

Given the distribution of wages, profit maximization of a bundler implies the demand function for each labor variety to satisfy

$$L_{1,t}(h) = \left[\frac{W_{1,t}(h)}{W_{1,t}} \right]^{-\frac{1+\theta^w}{\theta^w}} L^d_{1,t}. \tag{A.14}$$

The zero-profit condition yields that the wage (paid for one unit of the aggregate labor services) is

$$W_{1,t} = \left[\int_0^1 W_{1,t}(h)^{-\frac{1}{\theta^w}} dh \right]^{-\theta^w}. \tag{A.15}$$

The labor unions price their labor service $L_{1,t}(h)$ using contracts as in Calvo (1983) to maximize profits

$$\max_{W_{1,t}(h)} E_t \sum_{j=0}^{\infty} (\xi^w)^j \Lambda_{1,t+j} \left[(1+\tau^w)\bar{\Pi}^j W_{1,t}(h) - \tilde{W}_{1,t+j} \right] L_{1,t+j}(h)$$
$$s.t.$$
$$L_{1,t}(h) = \left[\frac{W_{1,t}(h)}{W_{1,t}} \right]^{-\frac{1+\theta^w}{\theta^w}} L^d_{1,t}.$$

The stochastic discount factor satisfies $\Lambda_{1,t+j} = \beta^j \frac{MU_{1,t+j}}{MU_{1,t}} \frac{P^c_{1,t}}{P^c_{1,t+j}}$. The first-order conditions associated with the union's maximization problem imply that the wage $W^*_{1,t}$ set in the current period by all reoptimizing unions satisfies

$$\frac{W^*_{1,t}}{P^d_{1,t}} = \frac{H^w_{1,t}}{G^w_{1,t}}, \tag{A.16}$$

with the definitions

$$\begin{aligned} H^w_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^w \beta)^j \frac{MU_{1,t+j}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\theta^w}{\theta^w} \frac{\tilde{W}_{1,t+j}}{P^d_{1,t+j}} \left[\frac{\left(\bar{\Pi}\right)^j W_{1,t}}{W_{1,t+j}} \right]^{-\frac{1+\theta^w}{\theta^w}} L^d_{1,t+j} \right\} \\ &= \frac{1+\theta^w}{\theta^w} \frac{\tilde{W}_{1,t}}{P^d_{1,t}} \left(\frac{1}{\Delta^w_{1,t}} L_{1,t} \right) + \xi^w \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} W_{1,t}}{W_{1,t+1}} \right)^{-\frac{1+\theta^w}{\theta^w}} H^w_{1,t+1} \right\} \end{aligned} \tag{A.17}$$

and

$$\begin{aligned} G^w_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^w \beta)^j \frac{MU_{1,t+j}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\tau^w}{\theta^w} \frac{\left(\bar{\Pi}\right)^j P^d_{1,t}}{P^d_{1,t+j}} \left[\frac{\left(\bar{\Pi}\right)^j W_{1,t}}{W_{1,t+j}} \right]^{-\frac{1+\theta^w}{\theta^w}} L^d_{1,t+j} \right\} \\ &= \frac{1+\tau^w}{\theta^w} \left(\frac{1}{\Delta^w_{1,t}} L_{1,t} \right) + \xi^w \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \frac{\bar{\Pi} P^d_{1,t}}{P^d_{1,t+1}} \left(\frac{\bar{\Pi} W_{1,t}}{W_{1,t+1}} \right)^{-\frac{1+\theta^w}{\theta^w}} G^w_{1,t+1} \right\} \end{aligned} \tag{A.18}$$

Wage inflation is defined as

$$\frac{W_{1,t}}{W_{1,t-1}} = \frac{W_{1,t}}{P^d_{1,t}} \frac{P^d_{1,t}}{P^d_{1,t-1}} \frac{P^d_{1,t-1}}{W_{1,t-1}}. \tag{A.19}$$

The definition of the wage index, the optimal wage, and the definition of wage inflation implies

$$(1-\xi^w)\left(\frac{W^*_{1,t}}{P^d_{1,t}}\frac{P^d_{1,t}}{W_{1,t}}\right)^{-\frac{1}{\theta^w}}+\xi^w\left(\bar{\Pi}\frac{W_{1,t-1}}{W_{1,t}}\right)^{-\frac{1}{\theta^w}}=1. \tag{A.20}$$

The fact that household labor supply and aggregate labor services are related via $L_{1,t}=\int_0^1 L_{1,t}(h)dh=\Delta^w_{1,t}L^d_{1,t}$ implies the dispersion in wages to satisfy

$$\Delta^w_{1,t} \;=\; (1-\xi^w)\left[\frac{W^*_{1,t}}{P^d_{1,t}}\frac{P^d_{1,t}}{W_{1,t}}\right]^{-\frac{1+\theta^w}{\theta^w}}+\xi^w\left[\bar{\Pi}\frac{W_{1,t-1}}{W_{1,t}}\right]^{-\frac{1+\theta^w}{\theta^w}}\Delta^w_{1,t-1}. \tag{A.21}$$

A.4 Production of Manufactured Goods

Intermediate variety producers introduce distinguishing characteristics into their output. Competitive bundlers sell the manufactured good $Y^d_{1,t}$ to households. It is

$$Y^d_{1,t}=\left[\int_0^1 Y_{1,t}(i)^{\frac{1}{1+\theta^p}}\,di\right]^{1+\theta^p}. \tag{A.22}$$

Given the distribution of prices, profit maximization of a bundler implies the demand function for each intermediate variety to satisfy

$$Y_{1,t}(i)=\left[\frac{P_{1,t}(i)}{P^d_{1,t}}\right]^{-\frac{1+\theta^p}{\theta^p}}Y^d_{1,t}. \tag{A.23}$$

The zero-profit condition yields that the price index is

$$P^d_{1,t}=\left[\int_0^1 P^d_{1,t}(i)^{-\frac{1}{\theta^p}}\,di\right]^{-\theta^p}. \tag{A.24}$$

Under our baseline assumptions, the foreign currency price of exports is $P^m_{2,t}=P^d_{1,t}/e_{1,t}$. Intermediate variety producers price their variety $Y_{1,t}(i)$ using contracts as in Calvo (1983). Firms experience country-specific technology shocks $z_{1,t}$

$$z_{1,t}=\rho^z z_{1,t-1}+\sigma^z\varepsilon^z_{1,t}. \tag{A.25}$$

With linear technology,

$$Y_{1,t}(i)=\exp(z_{1,t})\,L^d_{1,t}(i), \tag{A.26}$$

real marginal production costs, which are identical across firms, are given by

$$\frac{MC_{1,t}}{P^d_{1,t}}=\frac{1}{\exp(z_{1,t})}\frac{W_{1,t}}{P^d_{1,t}} \tag{A.27}$$

and the profit maximization problem is

$$\max_{P_{1,t}(i),\{Y_{1,t+j}(i)\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\xi^p)^j \Lambda_{1,t+j} \left((1+\tau^p)\,\bar{\Pi}^j P_{1,t}(i) - \frac{W_{1,t+j}}{\exp\left(z_{1,t+j}\right)} \right) Y_{1,t+j}(i)$$

$$s.t.$$

$$Y_{1,t+j}(i) = \left[\frac{P_{1,t+j}(i)}{P^d_{1,t+j}} \right]^{-\frac{1+\theta^p}{\theta^p}} Y^d_{1,t+j}. \tag{A.28}$$

The first-order conditions from the intermediate producer's maximization problem imply that the price $P^{d*}_{1,t}$ set in the current period by all reoptimizing producers satisfies

$$\frac{P^{d*}_{1,t}}{P^d_{1,t}} = \frac{H^p_{1,t}}{G^p_{1,t}}, \tag{A.29}$$

with the definitions

$$\begin{aligned} H^p_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^p \beta)^j \frac{MU_{1,t+j}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\theta^p}{\theta^p} \frac{MC_{1,t+j}}{P^d_{1,t+j}} \left[\frac{\left(\bar{\Pi}\right)^j P^d_{1,t}}{P^d_{1,t+j}} \right]^{-\frac{1+\theta^p}{\theta^p}} Y^d_{1,t+j} \right\} \\ &= \frac{1+\theta^p}{\theta^p} \frac{MC_{1,t}}{P^d_{1,t}} Y^d_{1,t} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} P^d_{1,t}}{P^d_{1,t+1}} \right)^{-\frac{1+\theta^p}{\theta^p}} H^p_{1,t+1} \right\}, \end{aligned} \tag{A.30}$$

where $\Lambda_{1,t+j} = \beta \frac{MU_{1,t+1}}{MU_{1,t}} \frac{P^c_{1,t}}{P^c_{1,t+j}}$ and

$$\begin{aligned} G^p_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^p \beta)^j \frac{MU_{1,t+j}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\tau^p}{\theta^p} \left[\frac{\left(\bar{\Pi}\right)^j P^d_{1,t}}{P^d_{1,t+j}} \right]^{1-\frac{1+\theta^p}{\theta^p}} Y^d_{1,t+j} \right\} \\ &= \frac{1+\tau^p}{\theta^p} Y^d_{1,t} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} P^d_{1,t}}{P^d_{1,t+1}} \right)^{1-\frac{1+\theta^p}{\theta^p}} G^p_{1,t+1} \right\}. \end{aligned} \tag{A.31}$$

The definition of the price index implies that

$$(1-\xi^p) \left(\frac{P^{d*}_{1,t}}{P^d_{1,t}} \right)^{\frac{-1}{\theta^p}} + \xi^p \left(\bar{\Pi} \frac{P^d_{1,t-1}}{P^d_{1,t}} \right)^{\frac{-1}{\theta^p}} = 1. \tag{A.32}$$

The fact that output and labor are related via $\int_0^1 \frac{Y_{1,t}(i)}{e^{z_{1,t}}} di = \int_0^1 L^d_{1,t}(di) di = L^d_{1,t}$

implies for the price dispersion measure $\Delta^p_{1,t}$ that

$$\Delta^p_{1,t} \;=\; (1-\xi^p)\left(\frac{P^{d*}_{1,t}}{P^d_{1,t}}\right)^{-\frac{1+\theta^p}{\theta^p}} + \xi^p\left(\bar{\Pi}\frac{P^d_{1,t-1}}{P^d_{1,t}}\right)^{-\frac{1+\theta^p}{\theta^p}}\Delta^p_{1,t-1}. \tag{A.33}$$

A.5 Market Clearing

Aggregating over households, market clearing for the domestic good requires

$$Y^d_{1,t} = C^d_{1,t} + M_{2,t} \tag{A.34}$$

where $M_{2,t}$ denotes the demand of the foreign country for the domestic good. Labor and product market differentiation imply that the labor supplied by household members is related to the output of the domestically produced good via

$$L_{1,t} = \Delta^w_{1,t}\Delta^p_{1,t}\frac{Y^d_t}{\exp\left(z_{1,t}\right)}. \tag{A.35}$$

Combining the two conditions yields

$$\exp\left(z_{1,t}\right)L_{1,t} = \Delta^w_{1,t}\Delta^p_{1,t}\left(C^d_{1,t} + M_{2,t}\right). \tag{A.36}$$

Domestically traded bonds are in zero net supply, requiring $D_{1,t+1|t} = 0$. For internationally traded bonds, market clearing requires

$$B_{11,t} + B_{21,t} = 0, \tag{A.37}$$

$$B_{12,t} + B_{22,t} = 0. \tag{A.38}$$

A.6 Useful Definitions and Simplifications

Under producer-currency-pricing, theory implies

$$\delta_{1,t} \;=\; \nu_{1,t} \tag{A.39}$$

$$\nu_{1,t} \;=\; \frac{1}{\nu_{2,t}}. \tag{A.40}$$

Defining the trade balance as

$$T_{1,t} \equiv e_t P^m_{2,t} M_{2,t} - P^m_{1,t} M_{1,t}, \tag{A.41}$$

we obtain the trade balance normalized by the value of exports as

$$\tilde{T}_{1,t} \;=\; \frac{T_{1,t}}{e_t P^m_{2,t} M_{2,t}} = 1 - \frac{1-\omega^c_1}{1-\omega^c_2}\left[\frac{\omega^c_1 \nu_{1,t}^{\frac{1}{\rho^c}} + (1-\omega^c_1)}{\omega^c_2 \nu_{2,t}^{\frac{1}{\rho^c}} + (1-\omega^c_2)}\right]^{-(1+\rho^c)}\delta_{1,t}\frac{C_{1,t}}{C_{2,t}}. \tag{A.42}$$

The optimal choices of the two bonds imply for country 1

$$\tilde{P}^b_{1,t} \equiv (1-\eta)\, P^b_{1,t} + \eta P^b_{2,t} = \phi^b_{1,t} \beta E_t \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{e_{1,t+1}}{e_{1,t}} \left\{ (1-\eta)\, \frac{e_{1,t}}{e_{1,t+1}} + \eta \right\} \tag{A.43}$$

and for country 2

$$\tilde{P}^b_{2,t} \equiv (1-\eta)\, P^b_{1,t} + \eta P^b_{2,t} = \phi^b_{2,t} \beta E_t \frac{\lambda_{2,t+1}}{\lambda_{2,t}} \left\{ (1-\eta)\, \frac{e_{1,t}}{e_{1,t+1}} + \eta \right\}, \tag{A.44}$$

where by construction

$$\tilde{P}^b_{1,t} = \tilde{P}^b_{2,t}. \tag{A.45}$$

The budget constraint of the household can be rewritten to record the evolution of the country's NFA position

$$\frac{1}{\phi^b_{1,t}} \tilde{P}^b_{2,t} \tilde{B}_{1,t} = \frac{\nu_{2,t-1}}{\nu_{2,t}} \frac{P^d_{2,t-1} M_{2,t-1}}{P^d_{2,t} M_{2,t}} \left\{ (1-\eta)\, \frac{e_{1,t-1}}{e_{1,t}} + \eta \right\} \tilde{B}_{1,t-1} + \tilde{T}_{1,t} \tag{A.46}$$

where we defined

$$\tilde{B}_{1,t} = \frac{\frac{e_{1,t} B_{12,t}}{\eta}}{e_{1,t} P^m_{2,t} M_{2,t}}.$$

The cost $\phi^b_{1,t}$ is given by

$$\phi^b_{1,t} = \exp\left(\phi^b_1 \tilde{B}_{1,t} \frac{\frac{\nu_{1,t}}{\delta_{1,t}} M_{2,t}}{C^d_{1,t} + \frac{\nu_{1,t}}{\delta_{1,t}} M_{2,t}} \right). \tag{A.47}$$

We define the consumption-price based real exchange rate as

$$rer_{1,t} = \left(\frac{F_{1,t}}{F_{2,t}} \right)^{\rho^c} \frac{\nu_{1,t}}{\delta_{1,t} \nu_{2,t}}. \tag{A.48}$$

A.7 Equilibrium Conditions for the Private Sector

For given sequences of the policy instruments set by the two policymakers, the endogenous variables have to satisfy the first-order and market-clearing conditions associated with the model laid out above. The full set of conditions describing the equilibrium conditions of the private sector consists of Equations (A.3), (A.5), (A.6), (A.9), (A.11), (A.12), (A.16), (A.17), (A.18), (A.19), (A.20), (A.21), (A.27), (A.29), (A.30), (A.31), (A.32), (A.33), (A.36), (A.43), (A.46), (A.47), for country 1 and their counterparts in the foreign country (not displayed) plus Equations (A.39), (A.40), (A.42), (A.45), (A.48). The exogenous shock process for technology in country 1 is given by Equation (A.25). A similar equation applies in country 2.

Let $\tilde{x}_t$ be the $(N-2) \times 1$ vector of endogenous variables excluding the policy instruments. The exogenous shocks are summarized in the vector ζ_t. Then, assuming that each country's central bank uses one instrument only, denoted $i_{1,t}$ and $i_{2,t}$ respectively, the $N-2$ first-order and market-clearing conditions of the model are summarized by

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0. \tag{A.49}$$

A.8 Model Variants

This section of the appendix describes the model variants that allow us to consider less-than-full pass-through of exchange rate movements to export prices, demand shocks, and alternative arrangements for international financial markets.

A.8.1 Local- and Dominant-Currency-Pricing

Under local-currency-pricing, we have to split the pricing problem of the firm by destination. The local-currency prices of the varieties are denoted by $P^d_{1,t}(i)$ and $P^m_{1,t}(i)$, respectively. The price indices

$$P^d_{1,t} = \left[\int_0^1 P^d_{1,t}(i)^{\frac{-1}{\theta^p}}\, di\right]^{-\theta^p}, \tag{A.50}$$

$$P^m_{1,t} = \left[\int_0^1 P^m_{1,t}(i)^{\frac{-1}{\theta^p}}\, di\right]^{-\theta^p} \tag{A.51}$$

evolve according to

$$(1-\xi^p)\left(\frac{P^{d*}_{1,t}}{P^d_{1,t}}\right)^{\frac{-1}{\theta^p}} + \xi^p\left(\bar{\Pi}\frac{P^d_{1,t-1}}{P^d_{1,t}}\right)^{\frac{-1}{\theta^p}} = 1, \tag{A.52}$$

$$(1-\xi^p)\left(\frac{P^{m*}_{1,t}}{P^m_{1,t}}\right)^{\frac{-1}{\theta^p}} + \xi^p\left(\bar{\Pi}\frac{\nu_{1,t-1}P^d_{1,t-1}}{\nu_{1,t}P^d_{1,t}}\right)^{\frac{-1}{\theta^p}} = 1 \tag{A.53}$$

in country 1 and similarly in country 2. Prices of firms that do not adjust optimally in the current period rise at the steady-state inflation rate of the country in which the goods are sold.

In addition, we define the dispersion measures

$$\begin{aligned}\Delta^d_{1,t} &= \int_0^1 \left[\frac{P^d_{1,t}(i)}{P^d_{1,t}}\right]^{-\frac{1+\theta^p}{\theta^p}} di \\ &= (1-\xi^p)\left(\frac{P^{d*}_{1,t}}{P^d_{1,t}}\right)^{-\frac{1+\theta^p}{\theta^p}} + \xi^p\left(\bar{\Pi}\frac{P^d_{1,t-1}}{P^d_{1,t}}\right)^{-\frac{1+\theta^p}{\theta^p}} \Delta^d_{1,t-1} \end{aligned} \tag{A.54}$$

and

$$\Delta_{1,t}^{m} = \int_0^1 \left[\frac{P_{1,t}^{m}(i)}{P_{1,t}^{m}}\right]^{-\frac{1+\theta^p}{\theta^p}} di$$
$$= (1-\xi^p)\left(\frac{P_{1,t}^{m*}}{P_{1,t}^{m}}\right)^{-\frac{1+\theta^p}{\theta^p}} + \xi^p\left(\bar{\Pi}\frac{P_{1,t-1}^{m}}{P_{1,t}^{m}}\right)^{-\frac{1+\theta^p}{\theta^p}}\Delta_{1,t-1}^{m}. \quad \text{(A.55)}$$

We continue to assume that the producers of the differentiated intermediate varieties purchase homogeneous input goods and differentiate them. The producers of the homogeneous input good use labor only as before

$$Y_{1,t}(i) = e^{z_{1,t}} L_{1,t}^{d}(i). \quad \text{(A.56)}$$

Aggregation implies

$$\int \frac{Y_{1,t}(i)}{e^{z_{1,t}}} di = \int L_{1,t}^{d}(i) di = L_{1,t}^{d}. \quad \text{(A.57)}$$

Market clearing for the intermediate varieties requires

$$\int Y_{1,t}(i) di = \int C_{1,t}^{d}(i) di + \frac{1}{\zeta_1}\int M_{2,t}(i) di. \quad \text{(A.58)}$$

or

$$\frac{e^{z_{1,t}}}{\Delta_{1,t}^{w}} L_{1,t}^{d} = \Delta_{1,t}^{d} C_{1,t}^{d} + \frac{1}{\zeta_1}\Delta_{2,t}^{m} M_{2,t}. \quad \text{(A.59)}$$

The value of total sales for firm i in period $t+j$ is

$$S_{1,t+j}(i) = \left(\bar{\Pi}\right)^{j} P_{1,t}^{d}(i) C_{1,t}^{d}(i) + \frac{1}{\zeta_1} e_t \left(\bar{\Pi}\right)^{j} P_{2,t}^{m}(i) M_{2,t}(i).$$

Expected discounted profits over the life-time of the given prices $P_{1,t}^{d}(i)$ and $P_{2,t}^{m}(i)$ are

$$E_t \sum_{j=0}^{\infty} (\xi^p)^j \Lambda_{1,t+j} \left\{ (1+\tau^p) S_{1,t+j}(i) - MC_{1,t+j}\left(C_{1,t+j}^{d}(i) + \frac{1}{\zeta_1} M_{2,t+j}(i)\right). \right\}$$

The conditions for the pricing of the domestically sold goods are unchanged subject to modifying the relevant demand driver from $Y_{1,t}^{d}$ to $C_{1,t}^{d}$. Hence, it is

$$\frac{P_{1,t}^{d*}}{P_{1,t}^{d}} = \frac{H_{1,t}^{p}}{G_{1,t}^{p}} \quad \text{(A.60)}$$

with the definitions

$$H_{1,t}^{p} = \frac{1+\theta^p}{\theta^p}\frac{MC_{1,t}}{P_{1,t}^{d}} C_{1,t}^{d} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P_{1,t}^{c}}{P_{1,t}^{d}}}{\frac{P_{1,t+1}^{c}}{P_{1,t+1}^{d}}} \left(\frac{\bar{\Pi} P_{1,t}^{d}}{P_{1,t+1}^{d}}\right)^{-\frac{1+\theta^p}{\theta^p}} H_{1,t+1}^{p} \right\}, \quad \text{(A.61)}$$

where $\Lambda_{1,t+j} = \beta \frac{MU_{1,t+1}}{MU_{1,t}} \frac{P^c_{1,t}}{P^c_{1,t+j}}$

$$G^p_{1,t} = \frac{1+\tau^p}{\theta^p} C^d_{1,t} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} P^d_{1,t}}{P^d_{1,t+1}} \right)^{1-\frac{1+\theta^p}{\theta^p}} G^p_{1,t+1} \right\}. \tag{A.62}$$

The conditions for export pricing imply

$$\frac{P^{m*}_{2,t}}{P^m_{2,t}} = \frac{MH^p_{1,t}}{MG^p_{1,t}} \tag{A.63}$$

with

$$\begin{aligned} MH^p_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^p \beta)^j \frac{MU_{1,t+j}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\theta^p}{\theta^p} \frac{MC_{1,t+j}}{P^d_{1,t+j}} \left[\frac{\bar{\Pi}^j P^m_{2,t}}{P^m_{2,t+j}} \right]^{-\frac{1+\theta^p}{\theta^p}} M_{2,t+j} \right\} \\ &= \frac{1+\theta^p}{\theta^p} \frac{MC_{1,t}}{P^d_{1,t}} M_{2,t} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} P^d_{2,t}}{P^d_{2,t+1}} \right)^{-\frac{1+\theta^p}{\theta^p}} MH^p_{1,t+1} \right\} \end{aligned} \tag{A.64}$$

$$\begin{aligned} MG^p_{1,t} &= \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{MU_{1,t}} E_t \left\{ \sum_{j=0}^{\infty} (\xi^p \beta)^j \frac{MU_{1,t+1}}{\frac{P^c_{1,t+j}}{P^d_{1,t+j}}} \frac{1+\tau^p}{\theta^p} \frac{\nu_{1,t+j}}{\delta_{1,t+j}} \left[\frac{\bar{\Pi}^j P^m_{2,t}}{P^m_{2,t+j}} \right]^{1-\frac{1+\theta^p}{\theta^p}} M_{2,t+j} \right\} \\ &= \frac{1+\tau^p}{\theta^p} \frac{\nu_{1,t}}{\delta_{1,t}} M_{2,t} + \xi^p \beta E_t \left\{ \frac{MU_{1,t+1}}{MU_{1,t}} \frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{1,t+1}}{P^d_{1,t+1}}} \left(\frac{\bar{\Pi} P^d_{2,t}}{P^d_{2,t+1}} \right)^{1-\frac{1+\theta^p}{\theta^p}} MG^p_{1,t+1} \right\}. \end{aligned} \tag{A.65}$$

Relative to the model with producer-currency-pricing, we newly introduce Equations (A.53), (A.55), (A.63), (A.64), (A.65), replace Equations (A.36), (A.30), (A.31) with Equations (A.59), (A.61), (A.62), and remove Equations (A.39), (A.40). We proceed analogously for country 2. Under dominant-currency-pricing, we make the above adjustments for only one country.

A.8.2 Valuation Shocks

We model valuation shocks as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016). We implement the valuation shock in the home country $\iota_{1,t}$ as follows

$$\mathcal{U}_{1,t} = E_t \sum_{j=0}^{\infty} \iota_{1,t+j} \beta^j \left\{ \ln \left(C_{1,t+j} - \kappa C_{1,t+j-1} \right) - \frac{\chi_0}{1+\chi} L^{1+\chi}_{1,t+j} \right\}, \tag{A.66}$$

with the growth of $\iota_{1,t}$ following an auto-regressive process of order 1

$$\ln\left(\frac{\iota_{1,t}}{\iota_{1,t-1}}\right) = \rho^{\iota}\ln\left(\frac{\iota_{1,t-1}}{\iota_{1,t-2}}\right) + \sigma^{\iota}\varepsilon^{\iota}_{1,t}. \tag{A.67}$$

Consequently, the first-order conditions of the household with respect to consumption and labor in Equations (A.5) and (A.9) need to be augmented as follows

$$MU_{1,t} = \left(\frac{\iota_{1,t}}{1-\kappa\frac{C_{1,t-1}}{C_{1,t}}} - \frac{\beta\kappa\iota_{1,t+1}}{\frac{C_{1,t+1}}{C_{1,t}}-\kappa}\right)\frac{1}{C_{1,t}}, \tag{A.68}$$

$$\frac{\tilde{W}_{1,t}}{P^d_{1,t}} = \iota_{1,t}\chi_{0,1}\frac{L^{\chi_1}_{1,t}}{MU_{1,t}}\frac{1}{F^{\rho^c}_{1,t}}. \tag{A.69}$$

A.8.3 Complete Markets and Financial Autarky

Under complete markets, we remove Equations (A.43), (A.46), (A.47), and their equivalent expressions for the foreign country, as well as Equations (A.42), (A.45) and newly introduce

$$\frac{MU_{1,t}}{MU_{2,t}} = e_0\frac{\lambda_{1,0}}{\lambda_{2,0}}\frac{\frac{P^c_{1,t}}{P^d_{1,t}}}{\frac{P^c_{2,t}}{P^d_{2,t}}}\frac{\nu_{2,t}}{\nu_{1,t}}\delta_{1,t} = \frac{1-\omega^c_1}{1-\omega^c_2}\left[\frac{\omega^c_1\nu^{\frac{1}{\rho^c}}_{1,t}+(1-\omega^c_1)}{\omega^c_2\nu^{\frac{1}{\rho^c}}_{2,t}+(1-\omega^c_2)}\right]^{-\rho^c}\delta_{1,t}, \tag{A.70}$$

where $e_0\frac{\lambda_{1,0}}{\lambda_{2,0}} = \frac{C_{2,0}}{C_{1,0}} = \frac{1-\omega^c_1}{1-\omega^c_2}$.

Under financial autarky, we remove Equations (A.43), (A.46), (A.47), and their equivalent expressions for the foreign country, as well as Equations (A.42), (A.45) and newly introduce

$$\frac{C_{2,t}}{C_{1,t}} = \frac{1-\omega^c_1}{1-\omega^c_2}\left[\frac{\omega^c_1\nu^{\frac{1}{\rho^c}}_{1,t}+(1-\omega^c_1)}{\omega^c_2\nu^{\frac{1}{\rho^c}}_{2,t}+(1-\omega^c_2)}\right]^{-(1+\rho^c)}\delta_{1,t}. \tag{A.71}$$

This condition is derived from the fact that trade is balanced at every point in time under financial autarky, i.e., $\delta_{1,t}M_{1,t} = M_{2,t}$.

A.9 Indirect Utility Function

Returning to our baseline model with international financial flows limited to non-state-contingent bonds and producer-currency-pricing, absent consumption habits ($\kappa = 0$), we can use the equations of our model to express household utility as an indirect function of relative prices, wage and price dispersion measures, and the net foreign asset position.

Recall that the definition of the trade balance, Equation A.42, implies

$$1-\tilde{T}_{1,t} = \frac{1-\omega^c_1}{1-\omega^c_2}\left(\frac{F_{1,t}}{F_{2,t}}\right)^{-(1+\rho^c)}\left(\frac{\nu_{1,t}}{\nu_{2,t}}\right)^{-\frac{1+\rho^c}{\rho^c}}\delta_{1,t}\frac{C_{1,t}}{C_{2,t}} \tag{A.72}$$

where we have made use of Equations A.3 and A.4.

Using the demand functions for good 1, the market clearing condition for good 1, Equation A.34, can be rewritten to obtain

$$\begin{aligned} Y_{1,t}^{d} &= C_{1,t}^{d} + M_{2,t} \\ &= \omega_1^c F_{1,t}^{-(1+\rho^c)} C_{1,t} + (1-\omega_2^c) F_{2,t}^{-(1+\rho^c)} \nu_{2,t}^{-\frac{1+\rho^c}{\rho^c}} C_{2,t} \\ &= \left[\omega_1^c + (1-\omega_1^c)\nu_{1,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{\delta_{1,t}}{1-\tilde{T}_{1,t}}\right] F_{1,t}^{-(1+\rho^c)} C_{1,t} \\ &= A_{1,t} C_{1,t}. \end{aligned} \tag{A.73}$$

Similarly, we obtain for the market clearing condition for good 2

$$\begin{aligned} Y_{2,t}^{d} &= C_{2,t}^{d} + M_{1,t} \\ &= \left[\omega_2^c + (1-\omega_2^c)\nu_{2,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{1-\tilde{T}_{1,t}}{\delta_{1,t}}\right] F_{2,t}^{-(1+\rho^c)} C_{2,t} \\ &= A_{2,t} C_{2,t}. \end{aligned} \tag{A.74}$$

From here on we use the short cuts $A_{1,t}$ and $A_{2,t}$, where

$$A_{1,t} = \left[\omega_1^c + (1-\omega_1^c)\nu_{1,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{\delta_{1,t}}{1-\tilde{T}_{1,t}}\right] F_{1,t}^{-(1+\rho^c)} \tag{A.75}$$

$$A_{2,t} = \left[\omega_2^c + (1-\omega_2^c)\nu_{2,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{1-\tilde{T}_{1,t}}{\delta_{1,t}}\right] F_{2,t}^{-(1+\rho^c)}. \tag{A.76}$$

Thus, the relationship between labor and production described in Equation A.35 yields for country 1

$$\begin{aligned} L_{1,t} &= \frac{\Delta_{1,t}^{w}\Delta_{1,t}^{p}}{\exp(z_{1,t})} \left[\omega_1^c + (1-\omega_1^c)\nu_{1,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{\delta_{1,t}}{1-\tilde{T}_{1,t}}\right] F_{1,t}^{-(1+\rho^c)} C_{1,t} \\ &= \frac{\Delta_{1,t}^{w}\Delta_{1,t}^{p}}{\exp(z_{1,t})} A_{1,t} C_{1,t} \end{aligned} \tag{A.77}$$

and for country 2

$$\begin{aligned} L_{2,t} &= \frac{\Delta_{2,t}^{w}\Delta_{2,t}^{p}}{\exp(z_{2,t})} \left[\omega_2^c + (1-\omega_2^c)\nu_{2,t}^{-\frac{1+\rho^c}{\rho^c}} \frac{1-\tilde{T}_{1,t}}{\delta_{1,t}}\right] F_{2,t}^{-(1+\rho^c)} C_{2,t} \\ &= \frac{\Delta_{2,t}^{w}\Delta_{2,t}^{p}}{\exp(z_{2,t})} A_{2,t} C_{2,t}. \end{aligned} \tag{A.78}$$

Finally, combining Equations A.5 and A.9 under the assumption of $\kappa = 0$ the consumption-labor trade-off allows us to express consumption in country 1 as a function of relative prices, dispersion measures, and, through the trade balance, of the net foreign asset position

$$C_{1,t} = \left(\frac{1}{\chi_0} \frac{\tilde{W}_{1,t}}{P^d_{1,t}} F^{\rho^c}_{1,t} \left(\frac{\Delta^w_{1,t} \Delta^p_{1,t}}{\exp(z_{1,t})} A_{1,t} \right)^{-\chi} \right)^{\frac{1}{1+\chi}}. \tag{A.79}$$

In turn, the finding for consumption implies for labor in country 1 that

$$L_{1,t} = \left(\frac{1}{\chi_0} \frac{\tilde{W}_{1,t}}{P^d_{1,t}} F^{\rho^c}_{1,t} \frac{\Delta^w_{1,t} \Delta^p_{1,t}}{\exp(z_{1,t})} A_{1,t} \right)^{\frac{1}{1+\chi}}. \tag{A.80}$$

The related expressions for country 2 are

$$C_{2,t} = \left(\frac{1}{\chi_0} \frac{\tilde{W}_{2,t}}{P^d_{2,t}} F^{\rho^c}_{2,t} \left(\frac{\Delta^w_{2,t} \Delta^p_{2,t}}{\exp(z_{2,t})} A_{2,t} \right)^{-\chi} \right)^{\frac{1}{1+\chi}} \tag{A.81}$$

$$L_{2,t} = \left(\frac{1}{\chi_0} \frac{\tilde{W}_{2,t}}{P^d_{2,t}} F^{\rho^c}_{2,t} \frac{\Delta^w_{2,t} \Delta^p_{2,t}}{\exp(z_{2,t})} A_{2,t} \right)^{\frac{1}{1+\chi}}. \tag{A.82}$$

Plugging the information provided by Equation A.79 and A.80 into the household utility function, we arrive at the indirect utility function for households in country 1

$$\begin{aligned} U_{1,t} &= \ln(C_{1,t}) - \frac{\chi_0}{1+\chi_1} L^{1+\chi}_{1,t} \\ &= \frac{1}{1+\chi} \ln\left(\frac{1}{\chi_0}\right) + \frac{1}{1+\chi} \ln\left(\frac{\tilde{W}_{1,t}}{P^d_{1,t}}\right) + \frac{1}{1+\chi} \ln\left(F^{\rho^c}_{1,t}\right) - \frac{\chi}{1+\chi} \ln\left(\frac{\Delta^w_{1,t}\Delta^p_{1,t}}{\exp(z_{1,t})} A_{1,t}\right) \\ &\quad - \frac{1}{1+\chi} \frac{\tilde{W}_{1,t}}{P^d_{1,t}} F^{\rho^c}_{1,t} \frac{\Delta^w_{1,t}\Delta^p_{1,t}}{\exp(z_{1,t})} A_{1,t} \end{aligned} \tag{A.83}$$

where $A_{1,t}$ and $F_{1,t}$ are functions of the relative prices $\nu_{1,t}$, $\delta_{1,t}$ and, through the trade balance, of the net foreign asset position.

B Appendix: Simple Model

To provide intuition for our findings through analytical results, we simplify our model along several dimensions.

1. We assume that up until period $T-1$ policymakers cooperate. In period T, policymakers can revisit whether to cooperate or not.
2. We also assume that period T is the last period of the model. Thus, all loans must be settled in this final period. Firms or households that are resetting prices or wages in period T set prices and wages only for this final period. With these assumptions in place, our analysis essentially reduces to a one-period model with inherited distributions for prices and wages and the net foreign asset position.
3. We consider the model with producer-currency-pricing, i.e., $\delta_{1,t} = \nu_{1,t}$ and $\nu_{1,t} = \frac{1}{\nu_{2,t}}$.
4. We make the following parameter choices:
   - To obtain a symmetric setting domestic and foreign parameters are all set at identical values, in particular it is $\omega_1^c = \omega_2^c$ and $\eta = \frac{1}{2}$.
   - To simplify the subsequent algebra and gain analytical tractability we set $\chi = 0$ which yields quasi-linearity of the utility function with respect to labor, we set $\tau^p = \theta^p$ and $\tau^w = \theta^w$ to remove distortions from monopolistic competition, and we exclude consumption habits by setting $\kappa = 0$.
5. Finally, we impose that wages are flexible ($\xi^w = 0$) and that the trade elasticity of substitution is unitary ($\rho^c = \infty$).

With the assumptions 1 through 4 in place, the wage- and price-setting equations A.16-A.18 and A.29-A.31, respectively, imply that newly set wages and prices satisfy

$$\frac{W_{1,T}^*}{P_{1,T}^d} = \frac{H_{1,T}^w}{G_{1,T}^w} = \frac{\tilde{W}_{1,T}}{P_{1,T}^d} = \tilde{w}_{1,T} \tag{B.1}$$

$$\frac{P_{1,T}^{d*}}{P_{1,T}^d} = \frac{H_{1,T}^p}{G_{1,T}^p} = mc_{1,T}. \tag{B.2}$$

The definitions of the price and wage levels imply that real marginal cost and wage inflation can be expressed as

$$mc_{1,T} = \left(\frac{1}{1-\xi^p} - \frac{\xi^p}{1-\xi^p} \left(\frac{\Pi_{1,T}}{\bar{\Pi}} \right)^{\frac{1}{\theta^p}} \right)^{-\theta^p}, \tag{B.3}$$

$$\tilde{w}_{1,T} = \left(\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w} \left(\frac{\Pi_{1,t}}{\bar{\Pi}} \frac{w_{1,t}}{w_{1,t-1}} \right)^{\frac{1}{\theta^w}} \right)^{-\theta^w} w_{1,t}. \tag{B.4}$$

Notice that Equation B.3 could also be used to back out inflation in terms of marginal cost only. The definition of the dispersion measures yield

$$\Delta_{1,t}^p = (1-\xi^p) \left(\frac{1}{1-\xi^p} - \frac{\xi^p}{1-\xi^p} \left(\frac{\Pi_{1,t}}{\bar{\Pi}} \right)^{\frac{1}{\theta^p}} \right)^{1+\theta^p} + \xi^p \left(\frac{\Pi_{1,t}}{\bar{\Pi}} \right)^{\frac{1+\theta^p}{\theta^p}} \Delta_{1,t-1}^p$$

(B.5)

$$\Delta^w_{1,t} = (1-\xi^w)\left(\frac{1}{1-\xi^w} - \frac{\xi^w}{1-\xi^w}\left(\frac{\Pi_{1,t}}{\bar{\Pi}}\frac{w_{1,t}}{w_{1,t-1}}\right)^{\frac{1}{\theta^w}}\right)^{1+\theta^w} + \xi^w\left(\frac{\Pi_{1,t}}{\bar{\Pi}}\frac{w_{1,t}}{w_{1,t-1}}\right)^{\frac{1+\theta^w}{\theta^w}}\Delta^w_{1,t-1} \tag{B.6}$$

where the real wage and real marginal costs are related via

$$w_{1,t} = \exp\left(z_{1,t}\right) mc_{1,t}. \tag{B.7}$$

Analogous derivations hold for country 2.

The consolidated household budget constraint/net foreign asset condition, Equation A.46, implies under the assumption that $\tilde{B}_{1,T} = 0$

$$\tilde{T}_{1,T} = -\frac{\delta_{1,T-1}^{\frac{1}{\rho^c}}}{\delta_{1,T}^{\frac{1}{\rho^c}}}\frac{F_{2,T-1}^{-(1+\rho^c)}C_{2,T-1}}{F_{2,T}^{-(1+\rho^c)}C_{2,T}}\left\{(1-\eta)\frac{\delta_{1,T-1}}{\delta_{1,T}}\frac{1}{\Pi_{1,T}} + \eta\frac{1}{\Pi_{2,T}}\right\}\tilde{B}_{1,T-1} \tag{B.8}$$

and the definition of the trade balance, Equation A.72, delivers

$$1-\tilde{T}_{1,T} = \frac{F_{1,T}^{-1}}{F_{2,T}^{-1}\delta_{1,T}^{-\frac{1}{1+\rho^c}}}\delta_{1,T}^{-\frac{1}{\rho^c}}\frac{\tilde{w}_{1,T}}{\delta_{1,T}\tilde{w}_{2,T}}. \tag{B.9}$$

For convenience, we also define the real trade balance, $\bar{T}_{1,t} = M_{2,t}\tilde{T}_{1,t}$, and the real net foreign asset position, $\bar{B}_{1,t-1} = M_{2,t-1}\tilde{B}_{1,t-1}$ (as opposed to relative to exports)

$$\bar{T}_{1,T} = \frac{1-\omega_1^c}{\chi_0}\left(\frac{\tilde{w}_{2,T}\delta_{1,T}}{F_{2,T}\delta_{1,T}^{-\frac{1}{\rho^c}}} - \frac{\tilde{w}_{1,T}}{F_{1,T}\delta_{1,T}^{\frac{1}{\rho^c}}}\right) \tag{B.10}$$

$$\bar{T}_{1,T} = -\frac{1}{2}\left\{\frac{1}{\Pi_{1,T}} + \frac{\delta_{1,T}}{\delta_{1,T-1}}\frac{1}{\Pi_{2,T}}\right\}\bar{B}_{1,T-1}. \tag{B.11}$$

Combining equations B.10 and B.11 delivers an expression for the terms of trade $\delta_{1,T}$ as a function of the net foreign asset position $\bar{B}_{1,T-1}$

$$\delta_{1,T} = \frac{\frac{\tilde{w}_{1,T}}{F_{1,T}\delta_{1,T}^{\frac{1}{\rho^c}}} - \frac{1}{\Pi_{1,T}}\frac{\chi_0}{1-\omega_1^c}\frac{\bar{B}_{1,T-1}}{2}}{\frac{\tilde{w}_{2,T}}{F_{2,T}\delta_{1,T}^{-\frac{1}{\rho^c}}} + \frac{1}{\delta_{1,T-1}}\frac{1}{\Pi_{2,T}}\frac{\chi_0}{1-\omega_1^c}\frac{\bar{B}_{1,T-1}}{2}}. \tag{B.12}$$

Using the real trade balance in the equations for consumption and labor, we find

$$C_{1,T} = \frac{1}{\chi_0}\tilde{w}_{1,T}F_{1,T}^{\rho^c} \tag{B.13}$$

$$L_{1,T} = \frac{\Delta^w_{1,T}\Delta^p_{1,T}}{\exp\left(z_{1,T}\right)}\left[\frac{\omega_1^c}{\chi_0}F_{1,T}^{-1}\tilde{w}_{1,T} + \frac{1-\omega_1^c}{\chi_0}F_{2,T}^{-1}\tilde{w}_{2,T}\delta_{1,T}^{\frac{1+\rho^c}{\rho^c}}\right] \tag{B.14}$$

$$C_{2,T} = \frac{1}{\chi_0}\tilde{w}_{2,T}F_{2,T}^{\rho^c} \tag{B.15}$$

$$L_{2,T} = \frac{\Delta^w_{2,T}\Delta^p_{2,T}}{\exp(z_{2,T})}\left[\frac{\omega_1^c}{\chi_0}F_{2,T}^{-1}\tilde{w}_{2,T} + \frac{1-\omega_1^c}{\chi_0}F_{1,T}^{-1}\tilde{w}_{1,T}\delta_{1,T}^{-\frac{1+\rho^c}{\rho^c}}\right]. \tag{B.16}$$

In this case, the indirect utility functions become functions of the choice $\Pi_{1,t}$ and $\Pi_{2,t}$ and the lagged (inherited) variables, in particular the net foreign asset position $\bar{B}_{1,T-1}$:

$$\begin{aligned} U_{1,T} &= \ln(C_{1,T}) - \chi_0 L_{1,T} \\ &= \ln\left(\frac{1}{\chi_0}\right) + \ln(\tilde{w}_{1,T}) + \ln\left(F_{1,T}^{\rho^c}\right) \\ &\quad - \frac{\Delta^w_{1,T}\Delta^p_{1,T}}{\exp(z_{1,T})}\left[\omega_1^c F_{1,T}^{-1}\tilde{w}_{1,T} + (1-\omega_1^c)F_{2,T}^{-1}\tilde{w}_{2,T}\delta_{1,T}^{\frac{1+\rho^c}{\rho^c}}\right] \end{aligned} \tag{B.17}$$

$$\begin{aligned} U_{2,T} &= \ln(C_{2,T}) - \chi_0 L_{2,T} \\ &= \ln\left(\frac{1}{\chi_0}\right) + \ln(\tilde{w}_{2,T}) + \ln\left(F_{2,T}^{\rho^c}\right) \\ &\quad - \frac{\Delta^w_{2,T}\Delta^p_{2,T}}{\exp(z_{2,T})}\left[\omega_1^c F_{2,T}^{-1}\tilde{w}_{2,T} + (1-\omega_1^c)F_{1,T}^{-1}\tilde{w}_{1,T}\delta_{1,T}^{-\frac{1+\rho^c}{\rho^c}}\right] \end{aligned} \tag{B.18}$$

where

$$F_{1,t} = \omega_1^c + (1-\omega_1^c)\delta_{1,t}^{-\frac{1}{\rho^c}} \tag{B.19}$$

$$F_{2,t} = \omega_2^c + (1-\omega_2^c)\delta_{1,t}^{\frac{1}{\rho^c}}. \tag{B.20}$$

$\delta_{1,T}$ is determined by Equation B.12. The variables $mc_{1,T}$, $\tilde{w}_{1,T}$, $w_{1,T}$, $\Delta^p_{1,T}$, $\Delta^w_{1,T}$ are determined by Equations B.3-B.7 given $\Pi_{1,T}$. The roles of $\Pi_{1,T}$ and $mc_{1,T}$ can be reversed, as done in the subsequent section, as long as some mild regularity conditions are satisfied.[33] Variables for country 2 are determined by analogous equations.

For the results that follow, we also impose the additional restrictions under point 5 above. With flexible wages ($\xi^w = 0$), the desired wage $\tilde{w}_{1,T}$ equals the real wage $w_{1,T}$. Both desired and real wages are proportional to real marginal costs $mc_{1,T}$. The unitary trade elasticity ($\rho^c = \infty$) simplifies the relationship between domestic and foreign real marginal costs. In this special case, we also set $z_{1,T} = z_{2,T} = 0$, $\delta_{1,T-1} = 1$, and $\Delta_{1,T-1} = \Delta_{2,T-1} = 1$.[34]

With these assumptions, the indirect utility functions of the households in coun-

[33] For the choice of monetary policy instrument between the real marginal cost and inflation to be equivalent, we need these two variables to map into each other one for one. As can be seen from Equation B.4, there will be a mapping between gross inflation and real marginal cost as long as net inflation is above -100% and as long as real marginal cost is non-negative.

[34] To a first-order approximation, the values of $z_{1,T}$, $z_{2,T}$ and $\delta_{1,T-1}$ have no effect on the subsequent analysis. The values of $\Delta_{1,T-1}$ and $\Delta_{2,T-1}$ have only small quantitative effects with no qualitative implications.

tries 1 and 2 become

$$U_{1,T} = \ln\left(\frac{1}{\chi_0}\right) + \ln(mc_{1,T}) - (1-\omega_1^c)\ln(\delta_{1,T}) - \Delta_{1,T}^p mc_{1,T} + (1-\omega_1^c)\Delta_{1,T}^p mc_{2,T}\left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T},\right) \tag{B.21}$$

$$U_{2,T} = \ln\left(\frac{1}{\chi_0}\right) + \ln(mc_{2,T}) + (1-\omega_1^c)\ln(\delta_{1,T}) - \Delta_{2,T}^p mc_{2,T} + (1-\omega_1^c)\Delta_{2,T}^p mc_{1,T}\left(\frac{mc_{2,T}}{mc_{1,T}} - \delta_{1,T}^{-1}\right). \tag{B.22}$$

Notice that, in turn, the terms of trade and the price dispersion measures can be expressed in terms of current real marginal costs and lagged endogenous variables. To this purpose, consider that the terms of trade can be expressed as

$$\delta_{1,T} = \frac{mc_{1,T} - \frac{1}{\Pi_{1,T}}\frac{\chi_0}{2(1-\omega_1^c)}\bar{B}_{1,T-1}}{mc_{2,T} + \frac{1}{\Pi_{2,T}}\frac{\chi_0}{2(1-\omega_1^c)}\bar{B}_{1,T-1}}. \tag{B.23}$$

In turn, price inflation and dispersion are only functions of real marginal costs

$$\frac{\Pi_{1,T}}{\bar{\Pi}} = \left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{1,T}^{-\frac{1}{\theta^p}}\right)^{\theta^p}, \tag{B.24}$$

$$\frac{\Pi_{2,T}}{\bar{\Pi}} = \left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{2,T}^{-\frac{1}{\theta^p}}\right)^{\theta^p}, \tag{B.25}$$

$$\Delta_{1,t}^p = (1-\xi^p) mc_{1,T}^{-\frac{1+\theta^p}{\theta^p}} + \xi^p\left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{1,T}^{-\frac{1}{\theta^p}}\right)^{1+\theta^p}\Delta_{1,t-1}^p, \tag{B.26}$$

$$\Delta_{2,t}^p = (1-\xi^p) mc_{2,T}^{-\frac{1+\theta^p}{\theta^p}} + \xi^p\left(\frac{1}{\xi^p} - \frac{1-\xi^p}{\xi^p} mc_{2,T}^{-\frac{1}{\theta^p}}\right)^{1+\theta^p}\Delta_{2,t-1}^p. \tag{B.27}$$

B.1 Cooperative Policies

We consider the optimal policies with and without cooperation. Under cooperation, the policymakers in the two countries choose real marginal costs (or monetary policy more generally) to maximize a common objective function. Without cooperation, the policymakers have country-specific objective functions. Letting real marginal costs be the policy instrument reduces algebraic complexities while being equivalent to choosing inflation as the policy instrument under mild regularity conditions, as spelled out earlier.

Using a general formulation, we write the policymakers' objective functions as $\alpha_1 U_{1,t} + (1-\alpha_1)U_{2,t}$ for country 1 and $\alpha_2 U_{1,t} + (1-\alpha_2)U_{2,t}$ for country 2. If the policymakers do not cooperate, we set $\alpha_1 = 1$, $\alpha_2 = 0$. If the policymakers cooperate, we set $\alpha_1 = \alpha_2 = \frac{1}{2}$.[35]

[35] Assigning identical objective functions to the two policymakers where each policymaker uses only one instru-

Assuming no uncertainty in period T, the optimization problem of the policymaker in country 1 is

$$\max_{mc_{1,T}} \{\alpha_1 U_{1,T} + (1-\alpha_1) U_{2,T}\} \tag{B.28}$$

which yields the first-order condition, or the implicit best-response function of country 1,

$$\begin{aligned} 0 \;=\; & \alpha_1 \Bigg\{ \frac{1}{mc_{1,T}} - (1-\omega_1^c) \frac{1}{\delta_{1,T}} \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} - \Delta_{1,T}^{p} - \Delta_{1,T}^{p'} mc_{1,T} \\ & + (1-\omega_1^c) \Delta_{1,T}' mc_{2,T} \left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T} \right) + (1-\omega_1^c) \Delta_{1,T} mc_{2,T} \left(\frac{1}{mc_{2,T}} - \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \right) \Bigg\} \\ & + (1-\alpha_1) \Bigg\{ (1-\omega_1^c) \frac{1}{\delta_{1,T}} \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \\ & + (1-\omega_1^c) \Delta_{2,T} \left(\frac{mc_{2,T}}{mc_{1,T}} - \delta_{1,T}^{-1} \right) + (1-\omega_1^c) \Delta_{2,T} mc_{1,T} \left(-\frac{mc_{2,T}}{mc_{1,T}^2} + \delta_{1,T}^{-2} \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \right) \Bigg\} \end{aligned} \tag{B.29}$$

With this premise, we can proceed to our first proposition.

Proposition 1 *In the absence of outstanding net foreign assets, the optimal cooperative monetary stance sets marginal costs equal to 1, so that the terms of trade are also equal to 1 (i.e., $\delta_{1,T} = 1$). Compared to this equilibrium, if the outstanding net foreign assets are nonzero, the monetary stance is tighter in the debtor country (the real marginal cost is below 1), and looser in the creditor country (the real marginal cost is above 1).*

Proof. Under cooperation, i.e., $\alpha_1 = \frac{1}{2}$, the first-order condition B.29 reduces to

$$\begin{aligned} 0 \;=\; & \frac{1}{mc_{1,T}} - \Delta_{1,T}^{p} - \Delta_{1,T}^{p'} mc_{1,T} \\ & + (1-\omega_1^c) \Delta_{1,T}' mc_{2,T} \left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T} \right) + (1-\omega_1^c) \Delta_{1,T} mc_{2,T} \left(\frac{1}{mc_{2,T}} - \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \right) \\ & + (1-\omega_1^c) \Delta_{2,T} \left(\frac{mc_{2,T}}{mc_{1,T}} - \delta_{1,T}^{-1} \right) + (1-\omega_1^c) \Delta_{2,T} mc_{1,T} \left(-\frac{mc_{2,T}}{mc_{1,T}^2} + \delta_{1,T}^{-2} \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \right) \end{aligned} \tag{B.30}$$

Absent debt, $\bar{B}_{1,T-1} = 0$, Equation B.33 implies that real marginal costs in both countries are equal to 1. Note that, absent debt, Equation B.23 implies $\delta_{1,T} = \frac{mc_{1,T}}{mc_{2,T}}$, so that Equation B.33 reduces to

$$0 \;=\; \frac{1}{mc_{1,T}} - \Delta_{1,T}^{p} - \Delta_{1,T}^{p'} mc_{1,T}. \tag{B.31}$$

ment delivers identical results as assigning the same objective function to one global policymaker that uses both these instruments.

Thus, the best-response function under cooperation for each country's policy choice of real marginal costs is independent of the other country's policy.[36] For $\Delta^p_{1,T-1} = 1$, setting real marginal costs $mc_{1,T}$ equal to 1 solves Equation B.31 since in this case price dispersion $\Delta^p_{1,T}$ is equal to 1, see Equation B.26, and the derivative of price dispersion $\Delta^{p'}_{1,T}$ is zero.[37] Analogously, country 2 optimally sets $mc_{2,T} = 1$. Accordingly the monetary policy stance under cooperation is symmetric. In turn, Equation B.23 implies $\delta_{1,T} = 1$. Quod erat demonstrandum.

Moving to the second part of the proposition, to assess the role of debt in influencing the choice of real marginal costs, we approximate the best response function B.29 and evaluate partial derivatives at the zero-debt cooperative equilibrium:

$$U_{mc_1,mc_1}\left(mc_{1,T} - mc^{co}_{1,T}\right) + U_{mc_1,\bar{B}_1}\left(\bar{B}_{1,T-1} - \bar{B}^{co}_{1,T-1}\right) = 0. \tag{B.32}$$

It is easy to show that to a first-order approximation, the equilibrium choice of $mc_{1,T}$ continues to be independent of the choice of $mc_{2,T}$ even in the presence of debt. This result implies that $U_{mc_1,mc_2} = 0$ and we omit it from Equation B.32. The coefficients U_{mc_1,mc_1} and $U_{mc_1,\bar{B}_1}$ satisfy

$$\begin{aligned}
U_{mc_1,mc_1} &= -\left(mc_{1,T}\right)^{-2} - 2\Delta^{p'}_{1,T} - \Delta^{p''}_{1,T} mc_{1,T} \\
&\quad + \left(1-\omega^c_1\right)\Delta''_{1,T} mc_{2,T}\left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T}\right) + \left(1-\omega^c_1\right)\Delta'_{1,T} mc_{2,T}\left(\frac{1}{mc_{2,T}} - \frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right) \\
&\quad + \left(1-\omega^c_1\right)\Delta'_{1,T} mc_{2,T}\left(\frac{1}{mc_{2,T}} - \frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right) + \left(1-\omega^c_1\right)\Delta_{1,T} mc_{2,T}\left(-\frac{\partial\delta^2_{1,T}}{\partial mc^2_{1,T}}\right) \\
&\quad + \left(1-\omega^c_1\right)\Delta_{2,T}\left(-\frac{mc_{2,T}}{mc^2_{1,T}} + \delta^{-2}_{1,T}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right) + \left(1-\omega^c_1\right)\Delta_{2,T}\left(-\frac{mc_{2,T}}{mc^2_{1,T}} + \delta^{-2}_{1,T}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right) \\
&\quad + \left(1-\omega^c_1\right)\Delta_{2,T} mc_{1,T}\left(-2\frac{mc_{2,T}}{mc^3_{1,T}} - 2\delta^{-3}_{1,T}\left(\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right)^2 + \delta^{-2}_{1,T}\frac{\partial\delta^2_{1,T}}{\partial mc^2_{1,T}}\right) \\
&= -\left(1 + \Delta^{p''}_{1,T}\right) = -\left(1 + \frac{1+\theta^p}{\theta^p}\frac{1-\xi^p}{\xi^p}\right) < 0,
\end{aligned} \tag{B.33}$$

$$\begin{aligned}
U_{mc_1,\bar{B}_1} &= -\left(1-\omega^c_1\right)\Delta^p_{1,T} mc_{2,T}\left(\frac{\Delta^{p'}_{1,T}}{\Delta^p_{1,T}}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} + \frac{\partial\delta^2_{1,T}}{\partial mc_{1,T}\bar{B}_{1,T-1}}\right) \\
&\quad + \left(1-\omega^c_1\right)\Delta^p_{2,T}\frac{mc_{1,T}}{\delta^2_{1,T}}\left(\frac{1}{mc_{1,T}}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} - \frac{2}{\delta_{1,T}}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} + \frac{\partial\delta^2_{1,T}}{\partial mc_{1,T}\bar{B}_{1,T-1}}\right) \\
&= -\left(1-\omega^c_1\right)\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} = \chi_0.
\end{aligned} \tag{B.34}$$

[36] This finding is independent of the value of the trade elasticity and the values of the country-specific technologies.

[37] If $\Delta^p_{1,T-1} > 1$—dispersion is bounded from below by 1—it is $mc_{1,T} < 1$ in the cooperative equilibrium since price dispersion $\Delta^p_{1,T}$ in Equation B.26 is minimized for $mc_{1,T} = \left(\frac{\xi_p + (1-\xi^p)\Delta^p_{1,T-1}}{\Delta^p_{1,T-1}}\right)^{\theta^p}$ and $\Delta^p_{1,T}$ is convex in $mc_{1,T}$.

Recall that at the point of approximation $mc_{1,T} = mc^{co}_{1,T} = 1$, $mc_{2,T} = mc^{co}_{2,T} = 1$, $\Delta^p_{1,T} = \Delta^p_{2,T} = 1$, $\Delta^{p'}_{1,T} = 0$, $\delta_{1,T} = \frac{mc_{1,T}}{mc_{2,T}} = 1$. In addition, the first and second derivatives of the terms of trade in Equation B.23 with regard to real marginal costs when evaluated at $\bar{B}_{1,T-1} = 0$ imply $\frac{\partial \delta_{1,T}}{\partial mc_{1,T}} = \frac{1}{mc_{2,T}}$ and $\frac{\partial \delta^2_{1,T}}{\partial mc^2_{1,T}} = 0$. Finally, the second derivative of the dispersion measure evaluated at the zero-debt equilibrium is $\Delta^{p''}_{1,T} = \Delta^{p''}_{2,T} = \frac{1+\theta^p}{\theta^p}\frac{1-\xi^p}{\xi^p}$ and the derivate of the terms of trade with respect to $\bar{B}_{1,T-1}$ at the zero-debt equilibrium is $\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}} = -\frac{\chi_0}{1-\omega^c_1}$. Note that country 1 is a creditor if $\bar{B}_{1,T-1} > 0$ and a debtor for $\bar{B}_{1,T-1} < 0$.

Thus, since $mc^{co}_{1,T} = 1$ and $\bar{B}^{co}_{1,T-1} = 0$, the first-order effect of debt on the equilibrium choice of $mc_{1,T}$ is

$$mc_{1,T} - 1 = \frac{\chi_0}{1 + \Delta^{p''}_1}\bar{B}_{1,T-1}. \tag{B.35}$$

Under cooperation, if country 1 is

- a debtor, i.e., $\bar{B}_{1,T-1} < 0$, its real marginal cost choice will be lower than in the zero-debt cooperative equilibrium;
- a creditor, i.e., $\bar{B}_{1,T-1} > 0$, its real marginal cost choice will be higher than in the zero-debt cooperative equilibrium.

These results prove the second part of the proposition. ■

As a corollary of our first proposition, notice that the introduction of debt also affects the terms of trade. To size the effects, first take into account that country 2 is a creditor (debtor) when country 1 is a debtor (creditor) and thus

$$mc_{2,T} - 1 = -\frac{\chi_0}{1 + \Delta^{p''}_1}\bar{B}_{1,T-1}. \tag{B.36}$$

Noting that absent debt $\delta_{1,T} = 1$, when there is debt the terms of trade change according to

$$\begin{aligned} \delta_{1,T} - 1 &= mc_{1,T} - mc_{2,T} - \frac{\chi_0}{1-\omega^c_1}\bar{B}_{1,T-1} \\ &= \left(2\frac{\chi_0}{1+\Delta^{p''}_1} - \frac{\chi_0}{1-\omega^c_1}\right)\bar{B}_{1,T-1}. \end{aligned} \tag{B.37}$$

The direct effect of a nonzero net foreign asset position, $-\frac{\chi_0}{1-\omega^c_1}\bar{B}_{1,T-1}$, pushes towards an improvement of the creditor's terms of trade and a worsening of the debtor' terms of trade. The indirect effect works through the change in relative marginal costs, $mc_{1,T} - mc_{2,T}$. Given Proposition 1, the marginal costs of the creditor country exceed the marginal costs of the debtor country. Thus, the indirect effect pushes towards a worsening of the creditor's terms of trade. The overall effect depends on the sign of the expression

$$2\frac{\chi_0}{1+\Delta^{p''}_1} - \frac{\chi_0}{1-\omega^c_1} = \chi_0\left(2\frac{\theta^p\xi^p}{1+\theta^p-\xi^p} - \frac{1}{1-\omega^c_1}\right), \tag{B.38}$$

which is negative under common parameter choices, in particular for sufficiently small markups θ^p. If country 1 is the creditor ($\bar{B}_{1,T-1} > 0$), its terms of trade improves compared to the zero-debt equilibrium although the improvement is dampened by its (relative) increase in real marginal costs. The monetary policy response in the cooperative equilibrium allocates some resources to the debtor country to ease its burden in repaying the inherited debt.

Notice that these findings apply qualitatively also if the inherited value of price dispersion $\Delta^p_{1,T-1}$ exceeds 1. Higher inherited price dispersion in a given country implies lower real marginal costs in that country in the cooperative equilibrium regardless of the level of net foreign assets:

$$mc_{1,T} - 1 = \frac{\chi_0}{1 + \Delta_1^{p''}} \bar{B}_{1,T-1} - \xi^p \left(\Delta^p_{1,T-1} - 1 \right). \tag{B.39}$$

B.2 Nationally Oriented, Non-Cooperative Policies

Proposition 2 *Relative to the cooperative equilibrium, in the non-cooperative equilibrium the monetary stance is contractionary in both countries but remains symmetric in the absence of debt. Debt induces an asymmetry in the non-cooperative monetary stance: the creditor's (debtor's) terms of trade improve (worsen) and the improvement (worsening) depends on the size of the net-foreign-asset position.*

Proof. Absent cooperation, i.e., $\alpha_1 = 1$, the first-order condition B.29 reduces to

$$\begin{aligned} 0 \;=\; & \frac{1}{mc_{1,T}} - \Delta^p_{1,T} - \Delta^{p'}_{1,T} mc_{1,T} - (1-\omega_1^c) \frac{1}{\delta_{1,T}} \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \\ & + (1-\omega_1^c) \Delta^{p'}_{1,T} mc_{2,T} \left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T} \right) + (1-\omega_1^c) \Delta^p_{1,T} mc_{2,T} \left(\frac{1}{mc_{2,T}} - \frac{\partial \delta_{1,T}}{\partial mc_{1,T}} \right). \end{aligned} \tag{B.40}$$

Absent debt, $\bar{B}_{1,T-1} = 0$, Equation B.40 implies that real marginal costs in both countries are less than 1. Note that absent debt Equation B.23 implies $\delta_{1,T} = \frac{mc_{1,T}}{mc_{2,T}}$, so that Equation B.40 reduces to

$$\frac{\omega_1^c}{mc_{1,T}} - \Delta^p_{1,T} - \Delta^{p'}_{1,T} mc_{1,T} = 0. \tag{B.41}$$

If Equation B.41 is evaluated at $mc_{1,T} = 1$, its left hand side equals $\omega_1^c - 1 < 0$. Since the left hand side of Equation B.41 coincides with the marginal utility of country 1 with respect to marginal costs, country 1's policymaker can raise national utility by setting marginal costs below 1. Given symmetry, the policymaker in country 2 also sets marginal costs below 1. Accordingly, in the absence of debt, monetary policy in the non-cooperative equilibrium is tighter than in the cooperative equilibrium but remains symmetric across countries, which proves the first part of the proposition.[38]

[38] This finding generalizes to other values of the trade elasticity as long as the elasticity is sufficiently above 0. This contractionary bias under non-cooperative policymaking has been documented in Corsetti and Pesenti (2001) and Benigno (2002).

Moving on to the second part of the proposition, to assess the role of debt for the equilibrium choice of real marginal costs, we approximate the best response function B.40 and evaluate the partial derivatives again at the zero-debt cooperative equilibrium (as opposed to the non-cooperative equilibrium):

$$U_{1,mc_1} + U_{1,mc_1,mc_1}\left(mc_{1,T} - mc_{1,T}^{co}\right) + U_{1,mc_1,\bar{B}_1}\left(\bar{B}_{1,T-1} - \bar{B}_{1,T-1}^{co}\right) = 0. \tag{B.42}$$

In contrast to Equation B.32, the approximation here features the additional term $U_{1,mc_1} \neq 0$ to reflect the fact that we do not approximate around the non-cooperative equilibrium. Therefore, Equation B.42 can also be used to measure the distance of the marginal cost choice without cooperation from the marginal cost choice under cooperation when $\bar{B}_{1,T-1} = 0$.

It is easy to show that to the first order the equilibrium choice of $mc_{1,T}$ does not depend on $mc_{2,T}$; the approximation term $U_{1,mc_1,mc_2} = 0$ and we omit it from Equation B.42. The coefficients U_{1,mc_1}, U_{1,mc_1,mc_1} and $U_{1,mc_1,\bar{B}_1}$ satisfy

$$U_{1,mc_1} = \frac{\omega_1^c}{mc_{1,T}} - \Delta_{1,T}^p - \Delta_{1,T}^{p'} mc_{1,T} = \omega_1^c - 1, \tag{B.43}$$

$$\begin{aligned}
U_{1,mc_1,mc_1} &= -\frac{1}{mc_{1,T}^2} - 2\Delta_{1,T}^{p'} - \Delta_{1,T}^{p''} mc_{1,T} \\
&\quad + (1-\omega_1^c)\frac{1}{\delta_{1,T}^2}\left(\frac{\partial \delta_{1,T}}{\partial mc_{1,T}}\right)^2 - (1-\omega_1^c)\frac{1}{\delta_{1,T}}\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T}^2} \\
&\quad + (1-\omega_1^c)\Delta_{1,T}^{p''} mc_{2,T}\left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T}\right) + (1-\omega_1^c)\Delta_{1,T}^{p'} mc_{2,T}\left(\frac{1}{mc_{2,T}} - \frac{\partial \delta_{1,T}}{\partial mc_{1,T}}\right) \\
&\quad + (1-\omega_1^c)\Delta_{1,T}^{p'} mc_{2,T}\left(\frac{1}{mc_{2,T}} - \frac{\partial \delta_{1,T}}{\partial mc_{1,T}}\right) - (1-\omega_1^c)\Delta_{1,T}^{p} mc_{2,T}\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T}^2} \\
&= -(\omega_1^c + \Delta_1^{p''}),
\end{aligned} \tag{B.44}$$

$$\begin{aligned}
U_{1,mc_1,\bar{B}_1} &= (1-\omega_1^c)\frac{1}{\delta_{1,T}^2}\frac{\partial \delta_{1,T}}{\partial mc_{1,T}}\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}} - (1-\omega_1^c)\frac{1}{\delta_{1,T}}\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T}\partial \bar{B}_{1,T-1}} \\
&\quad - (1-\omega_1^c)\Delta_{1,T}^{p'} mc_{2,T}\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}} - (1-\omega_1^c)\Delta_{1,T}^{p} mc_{2,T}\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T}\partial \bar{B}_{1,T-1}} \\
&= (1-\omega_1^c)\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}} = -\chi_0\frac{1-\xi^p}{\xi^p}.
\end{aligned} \tag{B.45}$$

Thus, since $mc_{1,T}^{co} = 1$ and $\bar{B}_{1,T-1}^{co} = 0$, we obtain

$$mc_{1,T} = \underbrace{1 - \frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}}}_{\text{mc choice without debt}} - \frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}}\bar{B}_{1,T-1}. \tag{B.46}$$

Absent debt, Equation B.46 provides an estimate for the distance between the non-cooperative choice of real marginal costs and the cooperative choice of real marginal costs—the contractionary bias is $mc_1^{nc} - mc_1^{co} = -\frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}}$. Expressed relative to the zero-debt choice of real marginal costs, country 1 sets $mc_{1,T}$ as a function of the net foreign asset position as

$$mc_1 - mc_1^{nc} = -\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} \bar{B}_{1,T-1}. \tag{B.47}$$

Without cooperation in the presence of debt, if country 1 is

- a debtor, i.e., $\bar{B}_{1,T-1} < 0$, its real marginal cost choice will be higher than in the zero-debt non-cooperative equilibrium;
- a creditor, i.e., $\bar{B}_{1,T-1} > 0$, its real marginal cost choice will be lower than in the zero-debt non-cooperative equilibrium.

Taking into account the fact that country 2 is a creditor (debtor) when country 1 is a debtor (creditor) implies

$$mc_2 - mc_2^{nc} = \frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} \bar{B}_{1,T-1}. \tag{B.48}$$

Notice that in the presence of debt the marginal cost choices of the creditor and debtor in the non-cooperative equilibrium move in the opposite direction from the respective choices in the cooperative equilibrium. As a result, the change in relative marginal costs will not dampen but reinforce the terms of trade improvement of the creditor country in the non-cooperative equilibrium according to

$$\begin{aligned} \delta_{1,T} - 1 &= mc_{1,T} - mc_{2,T} - \frac{\chi_0}{1-\omega_1^c} \bar{B}_{1,T-1} \\ &= -\left(2\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} + \frac{\chi_0}{1-\omega_1^c} \right) \bar{B}_{1,T-1}, \end{aligned} \tag{B.49}$$

where $-\left(2\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} + \frac{\chi_0}{1-\omega_1^c} \right)$ is always negative. As a result, the terms of trade improvement for the creditor country is more sizeable in the non-cooperative equilibrium than in the cooperative equilibrium for given size of the net foreign asset position, as the difference between Equation B.49 and B.37 is

$$-2\left(\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} + \frac{\chi_0}{1 + \Delta_1^{p''}} \right) \bar{B}_{1,T-1}. \tag{B.50}$$

The monetary policy response in the non-cooperative equilibrium inefficiently shifts resources away from the debtor country and thereby aggravates the debtor's burden in repaying the inherited debt. These results prove the second part of the proposition.

■

As a corollary of our second theorem, as in the case of the cooperative equilibrium, a higher inherited price dispersion in a given country implies lower real marginal costs in that country in the non-cooperative equilibrium regardless of the level of net foreign assets:

$$mc_{1,T} - mc_1^{nc} = -\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} \bar{B}_{1,T-1} - \xi^p \left(1 + \frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}}\right) \left(\Delta_{1,T-1}^p - 1\right). \quad \text{(B.51)}$$

The effect of a given value of the inherited price dispersion on the equilibrium choice of the real marginal costs is slightly stronger absent cooperation than with cooperation.

Finally, note that to remove the contractionary bias stemming from non-cooperative monetary policies, we could impose additional constraints on the policymakers as in Benigno and Benigno (2006) to implement policy from a timeless perspective. Let the policymaker of country 1 (and analogously for country 2) solve the augmented maximization problem

$$\max_{mc_{1,T}} U_{1,T} - \lambda_{1,T-1} \xi^p \pi_{1,T}^{\frac{1+\theta^p}{\theta^p}} mc_{1,T} - \lambda_{2,T-1} \xi^p \pi_{1,T}^{\frac{1}{\theta^p}},$$

where $\pi_{1,T}$ and $\Delta_{1,T}^p$ are functions of $mc_{1,T}$. If the Lagrange multipliers are set such that $\lambda_{1,T-1} = \omega_1^c - 1$ and $\lambda_{2,T-1} = 1 - \omega_1^c$, the contractionary bias is removed. Even with the systematic bias gone, however, the subsequent results about the gains from cooperation with nonzero debt hold.[39] Hence, we opted not to change the original formulation of the problem (equivalently setting $\lambda_{1,T-1} = \lambda_{2,T-1} = 0$). In the infinite horizon model, the non-cooperative deterministic steady state under commitment with zero-debt (to which the economy will converge in the long run) features no contractionary bias regardless of the existence of a contractionary bias in the period of the switch; the contractionary bias plays an insignificant role in explaining the gains from cooperation under commitment.

B.3 A Second-Order Approximation of the Global Welfare Function

To assess the gains from cooperation, we first derive an approximation of the global welfare function. Second, we evaluate the approximated welfare function at the cooperative and the non-cooperative equilibrium choices of real marginal costs to show that the gains from cooperation are increasing in the net foreign asset position, $\bar{B}_{1,T-1}$.

Proposition 3 *The (purely) quadratic approximation to the global welfare function around the symmetric cooperative equilibrium with zero debt (and $\Delta_{1,T-1} = \Delta_{2,T-1} = 1$) is given by*

$$\begin{aligned} U_T \approx\ & \bar{U} - \frac{1}{4}\left(1 + \Delta_1^{p''}\right)\left(mc_{1,T} - 1\right)^2 - \frac{1}{4}\left(1 + \Delta_2^{p''}\right)\left(mc_{2,T} - 1\right)^2 \\ & + \frac{\chi_0}{2}\left(mc_{1,T} - 1\right)\bar{B}_{1,T-1} - \frac{\chi_0}{2}\left(mc_{2,T} - 1\right)\bar{B}_{1,T-1} - \frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2. \end{aligned} \quad \text{(B.52)}$$

[39] The constraints in the augmented maximization problem stem from the forward-looking terms associated with the firms' price setting problem.

Proof. To prove this Proposition, we start with the global welfare function (under the restriction that $\alpha_1 = \alpha_2 = \frac{1}{2}$), which is given by

$$\begin{aligned} U_T &= \frac{1}{2}U_{1,T} + \frac{1}{2}U_{2,T} \\ &= \ln\left(\frac{1}{\chi_0}\right) + \frac{1}{2}\left(\ln\left(mc_{1,T}\right) - \Delta^p_{1,T}mc_{1,T} + \left(1-\omega^c_1\right)\Delta^p_{1,T}mc_{2,T}\left[\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T}\right]\right) \\ &\quad + \frac{1}{2}\left(\ln\left(mc_{2,T}\right) - \Delta^p_{2,T}mc_{2,T} + \left(1-\omega^c_1\right)\Delta^p_{2,T}mc_{1,T}\left[\frac{mc_{2,T}}{mc_{1,T}} - \delta^{-1}_{1,T}\right]\right). \quad \text{(B.53)} \end{aligned}$$

We take a second-order expansion of the global welfare function around the zero-debt cooperative equilibrium with global welfare $\bar{U}$.[40] Taking into account the fact that, with zero debt, the terms of trade $\delta_{1,T}$ equal the ratio of real marginal costs, we obtain

$$\begin{aligned} U_T \approx\; & \bar{U} + \frac{1}{2}\left\{\frac{1}{mc_{1,T}} - \Delta^p_{1,T} - \Delta^{p'}_{1,T}mc_1\right\}\Bigg|_{co}\left(mc_{1,T} - mc^{co}_1\right) \\ & + \frac{1}{2}\left\{\frac{1}{mc_{2,T}} - \Delta^p_{2,T} - \Delta^{p'}_{2,T}mc_{2,T}\right\}\Bigg|_{co}\left(mc_{2,T} - mc^{co}_2\right) \\ & - \frac{1-\omega^c_1}{2}\left\{\Delta_{1,T}mc_{2,T}\frac{\partial\delta_{1,T}}{\bar{B}_{1,T-1}} + \Delta_{2,T}mc_{1,T}\frac{\partial\delta^{-1}_{1,T}}{\bar{B}_{1,T-1}}\right\}\Bigg|_{co}\bar{B}_{1,T-1} \\ & + \frac{1}{4}\left\{-\frac{1}{mc_{1,T}} - 2\Delta^{p'}_{1,T} - \Delta^{p''}_{1,T}mc_{1,T} - \left(1-\omega^c_1\right)\Delta^p_{1,T}mc_{2,T}\frac{\partial^2\delta_{1,T}}{\partial mc^2_{1,T}}\right\}\Bigg|_{co}\left(mc_{1,T} - mc^{co}_1\right)^2 \\ & + \frac{1}{4}\left\{-\frac{1}{mc_{2,T}} - 2\Delta^{p'}_{2,T} - \Delta^{p''}_{2,T}mc_{2,T} - \left(1-\omega^c_1\right)\Delta^p_{2,T}mc_{1,T}\frac{\partial^2\delta_{2,T}}{\partial mc^2_{2,T}}\right\}\Bigg|_{co}\left(mc_{2,T} - mc^{co}_2\right)^2 \\ & - \frac{1-\omega^c_1}{4}\left\{\Delta_{1,T}mc_{2,T}\frac{\partial^2\delta_{1,T}}{\bar{B}^2_{1,T-1}} + \Delta_{2,T}mc_{1,T}\frac{\partial^2\delta^{-1}_{1,T}}{\bar{B}^2_{1,T-1}}\right\}\Bigg|_{co}\bar{B}^2_{1,T-1} \\ & - \frac{1-\omega^c_1}{2}\left\{\Delta^{p'}_{1,T}mc_{2,T}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} + \Delta^p_{1,T}mc_{2,T}\frac{\partial^2\delta_{1,T}}{\partial mc_{1,T}\partial\bar{B}_{1,T-1}}\right. \\ & \qquad \left. + \Delta^p_{2,T}\frac{\partial\delta^{-1}_{1,T}}{\partial\bar{B}_{1,T-1}} + \Delta^p_{2,T}mc_{1,T}\frac{\partial^2\delta^{-1}_{1,T}}{\partial mc_{1,T}\partial\bar{B}_{1,T-1}}\right\}\Bigg|_{co}\left(mc_{1,T} - mc^{co}_1\right)\bar{B}_{1,T-1} \\ & - \frac{1-\omega^c_1}{2}\left\{\Delta^{p'}_{2,T}mc_{1,T}\frac{\partial\delta^{-1}_{1,T}}{\partial\bar{B}_{1,T-1}} + \Delta^p_{2,T}mc_{1,T}\frac{\partial^2\delta^{-1}_{1,T}}{\partial mc_{2,T}\partial\bar{B}_{1,T-1}}\right. \\ & \qquad \left. + \Delta^p_{1,T}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}} + \Delta^p_{1,T}mc_{2,T}\frac{\partial^2\delta_{1,T}}{\partial mc_{2,T}\partial\bar{B}_{1,T-1}}\right\}\Bigg|_{co}\left(mc_{2,T} - mc^{co}_2\right)\bar{B}_{1,T-1}. \end{aligned} \quad \text{(B.54)}$$

[40] We maintain the assumptions that $z_{1,T} = z_{2,T} = 0$ and $\Delta^p_{1,T-1} = \Delta^p_{2,T-1} = 1$. Thus the zero-debt cooperative equilibrium features $mc_{1,T} = mc_{2,T} = 1$, $\Pi_{1,T} = \Pi_{2,T} = 1$, $\Delta^p_{1,T} = \Delta^p_{2,T} = 1$, $\Delta^{p'}_{1,T} = \Delta^{p'}_{2,T} = 0$.

The terms in each set of curly brackets are evaluated at the symmetric cooperative equilibrium without debt, as indicated by $\Big|_{co}$. Note that, from optimality conditions, the terms $\frac{1}{mc_{i,T}} - \Delta^{p}_{i,T} - \Delta^{p'}_{i,T} mc_{i,T}$ equal zero for $i = \{1, 2\}$. The relevant partial derivatives are

$$\frac{\partial \delta_{1,T}}{\partial mc_{1,T}} = \frac{1}{mc_{2,T}} = 1$$
$$\frac{\partial \delta_{1,T}}{\partial mc_{2,T}} = -\frac{mc_{1,T}}{mc_{2,T}^2} = -1$$
$$\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}} = -\frac{\frac{\chi_0}{1-\omega_1^c}}{\Pi_{1,T} mc_{2,T}} = -\frac{\chi_0}{1-\omega_1^c}$$
$$\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T} \partial \bar{B}_{1,T-1}} = \frac{1}{2} \frac{\frac{\chi_0}{1-\omega_1^c}}{mc_{2,T}^2} \left(\frac{\Pi'_{1,T} mc_{2,T}}{\Pi_{1,T}^2} - \frac{1}{\Pi_{2,T}} \right) = \frac{1}{2} \frac{\chi_0}{1-\omega_1^c} \left(\frac{1-\xi^p}{\xi^p} - 1 \right)$$
$$\frac{\partial^2 \delta_{1,T}}{\partial mc_{2,T} \partial \bar{B}_{1,T-1}} = \frac{1}{2} \frac{\frac{\chi_0}{1-\omega_1^c}}{mc_{2,T}^2} \left(\frac{\Pi'_{2,T} mc_{2,T}}{\Pi_{2,T}^2} + \frac{3}{\Pi_{2,T}} \right) = \frac{1}{2} \frac{\chi_0}{1-\omega_1^c} \left(\frac{1-\xi^p}{\xi^p} + 3 \right)$$
$$\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T}^2} = 0$$
$$\frac{\partial^2 \delta_{1,T}}{\partial \bar{B}_{1,T-1}^2} = \left(\frac{\frac{\chi_0}{1-\omega_1^c}}{\Pi_{1,T} mc_{2,T}} \right)^2 = \left(\frac{\chi_0}{1-\omega_1^c} \right)^2$$
$$\frac{\partial \delta_{1,T}^{-1}}{\partial \bar{B}_{1,T-1}} = -\frac{\partial \delta_{1,T}}{\partial \bar{B}_{1,T-1}}$$
$$\frac{\partial^2 \delta_{1,T}^{-1}}{\partial \bar{B}_{1,T-1}^2} = \frac{\partial^2 \delta_{1,T}}{\partial \bar{B}_{1,T-1}^2}$$
$$\frac{\partial^2 \delta_{1,T}^{-1}}{\partial mc_{1,T} \partial \bar{B}_{1,T-1}} = -\frac{\partial^2 \delta_{1,T}}{\partial mc_{2,T} \partial \bar{B}_{1,T-1}}$$
$$\frac{\partial^2 \delta_{1,T}^{-1}}{\partial mc_{2,T}^2} = 0$$
$$\frac{\partial^2 \delta_{1,T}^{-1}}{\partial mc_{2,T} \partial \bar{B}_{1,T-1}} = -\frac{\partial^2 \delta_{1,T}}{\partial mc_{1,T} \partial \bar{B}_{1,T-1}}.$$

Thus the approximation to the global utility function can be written as:

$$\begin{aligned} U_T \approx \; & \bar{U} - \frac{1}{4} \left(1 + \Delta_1^{p''} \right) (mc_{1,T} - 1)^2 - \frac{1}{4} \left(1 + \Delta_2^{p''} \right) (mc_{2,T} - 1)^2 \\ & + \frac{\chi_0}{2} (mc_{1,T} - 1) \bar{B}_{1,T-1} - \frac{\chi_0}{2} (mc_{2,T} - 1) \bar{B}_{1,T-1} - \frac{1}{2} \frac{\chi_0^2}{1-\omega_1^c} \bar{B}_{1,T-1}^2, \end{aligned} \tag{B.55}$$

which completes the proof of the proposition. ■

B.4 Assessing the Gains from Cooperation

We can use Equation B.55 to compute global welfare both under cooperation and without cooperation for nonzero debt levels.

Proposition 4 *The (purely) quadratic approximation to the welfare gains from cooperation are increasing in the size of the net foreign asset position:*

$$DDU_T^{co,nc} = \frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0}{1+\Delta_1^{p''}} - \frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\right)^2 \bar{B}_{1,T-1}^2 > 0, \tag{B.56}$$

where $DDU_T^{co,nc} = \left(U_T^{co} - \bar{U}\right) - \left(U_T^{nc} - \bar{U}^*\right)$ *is the second-order approximation of the difference in the change of global welfare from the steady state.*

Proof. We proceed in three steps. First, we use Proposition 3 to obtain the second-order approximation to the welfare function under cooperative policies. Second, using the same proposition, we derive the approximate welfare function under non-cooperative policies. Third we sign the difference in the welfare under these alternative policies.

Global Welfare Under Cooperation. Under cooperation, Equation B.35 implies $mc_{1,T} - 1 = \frac{\chi_0}{1+\Delta_1^{p''}}\bar{B}_{1,T-1}$ and similarly for country 2 (with opposite sign); using this relation in the approximated global welfare function to substitute out real marginal costs

$$U_T^{co} - \bar{U} \approx -\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 + \frac{1}{2}\frac{\chi_0^2}{1+\Delta_1^{p''}}\bar{B}_{1,T-1}^2. \tag{B.57}$$

Overall, global welfare is always reduced when the net foreign asset position is nonzero because $1+\Delta_1^{p''} = 1 + \frac{1+\theta_p}{\theta_p}\frac{1-\xi^p}{\xi^p} > 1$. However, the terms of trade response in the cooperative equilibrium buffers the decline in global welfare as captured by the second term. In the cooperative equilibrium with debt the creditor (debtor) runs easier (tighter) monetary policy relative to the zero-debt equilibrium; this monetary policy response restrains the improvement of the creditor's terms of trade caused by the presence of debt (relative to keeping marginal costs unchanged). This policy improves global efficiency as it shifts resources to the debtor country to ease its burden in repaying the inherited debt.

Global Welfare Absent Cooperation. Absent cooperation, Equation B.47 allows us to simplify the approximated global welfare function to

$$\begin{aligned} U_T^{nc} \approx\ & \bar{U} - \frac{1}{4}\left(1+\Delta_1^{p''}\right)\left(-\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}} - \frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1}\right)^2 \\ & -\frac{1}{4}\left(1+\Delta_1^{p''}\right)\left(-\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}} + \frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1}\right)^2 \end{aligned}$$

$$
\begin{aligned}
&+\frac{\chi_0}{2}\left(-\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}-\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1}\right)\bar{B}_{1,T-1} \\
&-\frac{\chi_0}{2}\left(-\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}+\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1}\right)\bar{B}_{1,T-1}-\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 \\
= {}& \bar{U}-\frac{1}{2}\left(1+\Delta_1^{p''}\right)\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)^2-\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 \\
&-\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(\chi_0+\frac{1}{2}\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(1+\Delta_1^{p''}\right)\right)\bar{B}_{1,T-1}^2. \qquad \text{(B.58)}
\end{aligned}
$$

Global welfare absent cooperation is lower than global welfare under cooperation when debt is zero and reaches $\bar{U}^*=\bar{U}-\frac{1}{2}\left(1+\Delta_1^{p''}\right)\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)^2$. Relative to the zero-debt non-cooperative equilibrium, a nonzero net foreign asset position always lowers global utility according to

$$
U_T^{nc}-\bar{U}^* = -\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2-\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(\chi_0+\frac{1}{2}\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(1+\Delta_1^{p''}\right)\right)\bar{B}_{1,T-1}^2. \qquad \text{(B.59)}
$$

The fact that, in the non-cooperative equilibrium with debt, the creditor (debtor) runs tighter (easier) monetary policy relative to the no-debt equilibrium—a policy that inefficiently reinforces the creditor's terms of trade improvement—amplifies the reduction in global welfare when the net foreign asset is not zero.

Gains from Cooperation To isolate the gains from cooperation as a function of the net foreign assets, we focus on the difference in the change of global welfare given by $DDU_T=\left(U_T^{co}-\bar{U}\right)-\left(U_T^{nc}-\bar{U}^*\right)$

$$
DDU_T^{co,nc} = \frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0}{1+\Delta_1^{p''}}-\frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\right)^2\bar{B}_{1,T-1}^2. \qquad \text{(B.60)}
$$

Accordingly, since the coefficient on $\bar{B}_{1,T-1}$ is positive, the gains are increasing in the overall size of the net foreign asset position, which completes the proof of the proposition. ■

Notice that the overall magnitude of the gains for a given net foreign asset position depends on the size of the relative terms of trade improvement (worsening) of the creditor (debtor) and therefore the fact that, in the presence of debt, creditors (debtors) move the policy instrument in opposite directions when comparing the cooperative with the non-cooperative equilibrium.

B.5 Inflation Targeting

Proposition 5 *While global welfare is lower under strict inflation targeting than under the optimized cooperative policies, it is higher than under non-cooperative policies. To a second-order approximation, the disadvantage (advantage) vis-à-vis the cooperative (non-cooperative) policies is increasing in the net foreign asset position.*

Proof. We compare (strict) inflation targeting to both the cooperative and the non-cooperative policies. Under strict inflation targeting, policymakers set $\Pi_{1,T} = \bar{\Pi}$ and $\Pi_{2,T} = \bar{\Pi}$ and thus $mc_{1,T} = mc_{2,T} = 1$ regardless of the net foreign asset position. Applying this insight to the approximated global welfare function, Equation B.55, yields

$$U_T^{it} - \bar{U} \approx -\frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2. \tag{B.61}$$

Under (strict) inflation targeting global welfare cannot be higher than under the optimal cooperative policy as inflation targeting restricts the adjustment of the terms of trade:

$$DDU_T^{co,it} = \left(U_T^{co} - \bar{U}\right) - \left(U_T^{it} - \bar{U}\right) \approx \frac{1}{2}\frac{\chi_0^2}{1+\Delta_1^{p''}}\bar{B}_{1,T-1}^2 > 0. \tag{B.62}$$

Absent debt, inflation targeting implements the optimal cooperative policy. Yet in the presence of debt, inflation targeting prevents the creditor from implementing somewhat easier monetary policy and the debtor from implementing slightly tighter monetary policy and thus fails to efficiently restrain the improvement of the creditor's terms of trade.

However, (strict) inflation targeting yields higher global welfare than nationally oriented (non-cooperative) policies:

$$\begin{aligned} DDU_T^{it,nc} &= \left(U_T^{it} - \bar{U}\right) - \left(U_T^{nc} - \bar{U}^*\right) \\ &\approx \frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}}\left(\chi_0 + \frac{1}{2}\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}}\left(1+\Delta_1^{p''}\right)\right)\bar{B}_{1,T-1}^2 \end{aligned} \tag{B.63}$$

where we again isolate the effects of the net foreign asset position. Inflation targeting brings about higher welfare than the non-cooperative polices as inflation targeting prevents the additional inefficient policy-induced terms of trade improvement (worsening) of the creditor (debtor) that occurs in the non-cooperative equilibrium.

■

B.6 Consumption, Labor and the Gains from Cooperation

We provide additional information on the response of consumption and labor across equilibria and the inherited level of debt on the bases of linear approximations (ignoring higher order terms) that is valid for small nonzero asset positions.

Proposition 6 *When considering the switch from cooperation to non-cooperation, the creditor country plans to reduce consumption at the benefit of increasing leisure (lower labor).*

Proof. Recall that with flexible wages and a unitary trade elasticity consumption and labor in country 1 satisfy

$$C_{1,T} = \frac{1}{\chi_0} mc_{1,T} \delta_{1,T}^{-\left(1-\omega_1^c\right)}, \tag{B.64}$$

$$L_{1,T} = \frac{1}{\chi_0} \Delta_{1,T}^p mc_{1,T} - \frac{1}{\chi_0}\left(1-\omega_1^c\right) \Delta_{1,T}^p mc_{2,T} \left(\frac{mc_{1,T}}{mc_{2,T}} - \delta_{1,T}\right). \tag{B.65}$$

Approximating these equations around the zero-debt cooperative equilibrium (with consumption and labor both equal to $\frac{1}{\chi_0}$) delivers

$$C_{1,T} \approx \frac{1}{\chi_0} + \frac{1}{\chi_0}\left(mc_{1,T}-1\right) - \frac{1-\omega_1^c}{\chi_0}\left(\delta_{1,T}-1\right), \tag{B.66}$$

$$L_{1,T} \approx \frac{1}{\chi_0} + \frac{1}{\chi_0}\left(mc_{1,T}-1\right) - \frac{1-\omega_1^c}{\chi_0}\left[\left(mc_{1,T}-1\right) - \left(mc_{2,T}-1\right) - \left(\delta_{1,T}-1\right)\right]. \tag{B.67}$$

We evaluate Equations B.66 and B.67 under two settings:

1. The cooperative equilibrium using Equations B.35, B.36 and B.37 yields:

$$C_{1,T} - \frac{1}{\chi_0} = \underbrace{\frac{2\omega_1^c + \Delta_1^{p''}}{1+\Delta_1^{p''}}}_{>1} \bar{B}_{1,T-1}, \tag{B.68}$$

$$L_{1,T} - \frac{1}{\chi_0} = \underbrace{-\frac{\Delta_1^{p''}}{1+\Delta_1^{p''}}}_{>-1} \bar{B}_{1,T-1}. \tag{B.69}$$

   These equations show that consumption increases with the asset position, whereas the amount of labor decreases. The terms of trade response in this case is

$$\delta_{1,T} - 1 = \chi_0 \left(\frac{2}{1+\Delta_1^{p''}} - \frac{1}{1-\omega_1^c}\right) \bar{B}_{1,T-1}. \tag{B.70}$$

2. The case if country 1 deviates while country 2 continues to act cooperatively ($\alpha_1 = 1$ and $\alpha_2 = \frac{1}{2}$) using Equations B.47 and B.36 for the respective marginal cost terms and the fact that $\delta_{1,T} - 1 = mc_{1,T} - mc_{2,T} - \frac{\chi_0}{1-\omega_1^c} \bar{B}_{1,T-1}$ yields:

$$C_{1,T} - \frac{1}{\chi_0} = -\frac{\omega_1^c}{\chi_0} \frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}} + \underbrace{\left(\frac{\omega_1^c + \Delta_1^{p''}}{1+\Delta_1^{p''}} - \frac{\omega_1^c \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}}\right)}_{<1} \bar{B}_{1,T-1} \tag{B.71}$$

$$L_{1,T} - \frac{1}{\chi_0} = -\frac{1}{\chi_0}\frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}} - \underbrace{\left(1 + \frac{\frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}}\right)}_{<-1} \bar{B}_{1,T-1}. \tag{B.72}$$

The terms of trade response in this case is

$$\delta_{1,T} - 1 = -\frac{1-\omega_1^c}{\omega_1^c + \Delta_1^{p''}} + \underbrace{\chi_0 \left(\frac{1}{1+\Delta_1^{p''}} - \frac{\frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} - \frac{1}{1-\omega_1^c}\right)}_{<0 \text{ for sufficiently high home bias (as in numerical model)}} \bar{B}_{1,T-1}. \tag{B.73}$$

Note that this approach is appropriate as the best-response functions for marginal costs for each country—both under cooperation and non-cooperation—do not depend on the marginal cost choice of the other country.

With home bias in consumption ($\omega_1^c > 0.5$) and country 1 being a creditor ($B_{1,T-1} \geq 0$), comparing the consumption terms, Equations B.69 and B.71, for given value of $B_{1,T-1}$ consumption in country 1 turns out to be lower when country 1 deviates from cooperation unilaterally. Comparing the labor terms, Equations B.69 and B.72, for given value of $B_{1,T-1}$ labor in country 1 turns out to be lower when country 1 deviates from cooperation unilaterally. Notice that this ranking of consumption and labor holds even abstracting from the intercept terms in Equations B.71 and B.72. The relative consumption and labor responses are also reflected in the fact that the creditor country's terms of trade strengthens by more when the creditor country deviates unilaterally. ■

C Appendix: State-Contingent Welfare Weights

Thus far, we have measured the global gains from cooperation under the assumption that the utility of each country has an equal weight in the global welfare function, i.e. $\alpha_1 = \frac{1}{2}$, reflecting the symmetry of the two countries in the initial steady state. We now generalize our results to allow for state-contingent welfare weights in the period for which we compute the gains from cooperation. These state-contingent weights reflect differences across countries that can arise away from the steady state. We pursue two alternative schemes for state-contingent weights: the Pareto approach and the Negishi approach.

Under the Pareto approach, we adjust the weights in the global welfare function so that both countries are at least as well off in the cooperative equilibrium as they would be in the non-cooperative equilibrium. Under the Negishi approach, the weights reflect the marginal utilities of wealth in the non-cooperative equilibrium in each country.

We begin by deriving a purely quadratic approximation of the global welfare function with arbitrary welfare weights, and compute the implied gains from cooperation. We then provide results specific to each of the two approaches.

C.1 Gains from Cooperation with State-Contingent Welfare Weights

Consider the generalized global welfare function with state-contingent weights, $\alpha_{1,T}U_{1,T}+(1-\alpha_{1,T})U_{2,t}$. To assess the gains from cooperation with weights, $\alpha_{1,T}$ and $(1-\alpha_{1,T})$, we focus again on a second-order approximation taken around the cooperative equilibrium with zero debt and equal weights. This time, however, the approximation reflects possible variation in the weights.

Proposition 7 *The (purely) quadratic approximation of the global welfare function around the cooperative equilibrium with zero debt and equal weights (and no price dispersion, i.e.,* $\Delta_{1,T-1} = \Delta_{2,T-1} = 1$*) is given by*

$$\begin{aligned} U_T - \bar{U} \approx\; & -\frac{1}{4}\left(1+\Delta_1^{p''}\right)(mc_{1,T}-1)^2 - \frac{1}{4}\left(1+\Delta_2^{p''}\right)(mc_{2,T}-1)^2 \\ & +\frac{\chi_0}{2}(mc_{1,T}-1)\bar{B}_{1,T-1} - \frac{\chi_0}{2}(mc_{2,T}-1)\bar{B}_{1,T-1} - \frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 \\ & -2(1-\omega_1^c)(mc_{1,T}-1)\,d\alpha_{1,T} + 2(1-\omega_1^c)(mc_{2,T}-1)\,d\alpha_{1,T} \\ & +4\chi_0\bar{B}_{1,T-1}d\alpha_{1,T}. \end{aligned} \tag{C.1}$$

where $d\alpha_{1,T} = \alpha_{1,T} - \frac{1}{2}$.

Proof. We rewrite the global welfare function in terms of the sum of the utility functions, $(U_{1,T}+U_{2,t})$, and their difference, $(U_{1,T}-U_{2,t})$, interacted with the departure from equal weights, $d\alpha_{1,T} = \alpha_{1,T} - \frac{1}{2} \neq 0$:

$$U_{1,T} = \alpha_{1,T}U_{1,T} + (1-\alpha_{1,T})U_{2,t} = \frac{1}{2}(U_{1,T}+U_{2,t}) + (U_{1,T}-U_{2,t})\,d\alpha_{1,T}. \tag{C.2}$$

The approximation to the first term in Equation C.2 is given by Proposition 3 in Section B.3 of the Appendix.

$$\begin{aligned}\frac{1}{2}\left(U_{1,T}+U_{2,T}\right) \approx\ & \bar{U}-\frac{1}{4}\left(1+\Delta_{1}^{p''}\right)\left(mc_{1,T}-1\right)^{2}-\frac{1}{4}\left(1+\Delta_{2}^{p''}\right)\left(mc_{2,T}-1\right)^{2} \\ & +\frac{\chi_{0}}{2}\left(mc_{1,T}-1\right)\bar{B}_{1,T-1}-\frac{\chi_{0}}{2}\left(mc_{2,T}-1\right)\bar{B}_{1,T-1}-\frac{1}{2}\frac{\chi_{0}^{2}}{1-\omega_{1}^{c}}\bar{B}_{1,T-1}^{2}.\end{aligned}$$

The second-order approximation for the second term in Equation C.2, $\left(U_{1,T}-U_{2,t}\right)d\alpha_{1,T}$, is

$$\begin{aligned}\left(U_{1,T}-U_{2,t}\right)d\alpha_{1,T} \approx\ & \left(U_{1,mc_1}-U_{2,mc_1}\right)\left(mc_{1,T}-1\right)d\alpha_{1,T} \\ & +\left(U_{1,mc_2}-U_{2,mc_2}\right)\left(mc_{2,T}-1\right)d\alpha_{1,T}+\left(U_{1,\bar{B}}-U_{2,\bar{B}}\right)\bar{B}_{1,T-1}d\alpha_{1,T} \\ =\ & \left[-2\left(1-\omega_{1}^{c}\right)\left(mc_{1,T}-1\right)+2\left(1-\omega_{1}^{c}\right)\left(mc_{2,T}-1\right)+4\chi_{0}\bar{B}_{1,T-1}\right]d\alpha_{1,T}\end{aligned}$$

since

$$\begin{aligned}U_{1,mc_1} =\ & \frac{1}{mc_{1,T}}-\left(1-\omega_{1}^{c}\right)\frac{1}{\delta_{1,T}}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}-\Delta_{1,T}^{p}-\Delta_{1,T}^{p'}mc_{1,T} \\ & +\left(1-\omega_{1}^{c}\right)\Delta_{1,T}'mc_{2,T}\left(\frac{mc_{1,T}}{mc_{2,T}}-\delta_{1,T}\right) \\ & +\left(1-\omega_{1}^{c}\right)\Delta_{1,T}mc_{2,T}\left(\frac{1}{mc_{2,T}}-\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right)=-\left(1-\omega_{1}^{c}\right) \\ U_{2,mc_1} =\ & \left(1-\omega_{1}^{c}\right)\frac{1}{\delta_{1,T}}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}} \\ & +\left(1-\omega_{1}^{c}\right)\Delta_{2,T}\left(\frac{mc_{2,T}}{mc_{1,T}}-\delta_{1,T}^{-1}\right) \\ & +\left(1-\omega_{1}^{c}\right)\Delta_{2,T}mc_{1,T}\left(-\frac{mc_{2,T}}{mc_{1,T}^{2}}+\delta_{1,T}^{-2}\frac{\partial\delta_{1,T}}{\partial mc_{1,T}}\right)=\left(1-\omega_{1}^{c}\right) \\ U_{1,mc_2} =\ & U_{2,mc_1} \\ U_{2,mc_2} =\ & U_{1,mc_1} \\ U_{1,\bar{B}_1} =\ & -(1-\omega_{1}^{c})\frac{1}{\delta_{1,T}}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}}-(1-\omega_{1}^{c})\Delta_{1,T}^{p}mc_{2,T}\frac{\partial\delta_{1,T}}{\partial\bar{B}_{1,T-1}}=2\chi_{0} \\ U_{2,\bar{B}_1} =\ & -U_{1,\bar{B}_1}.\end{aligned}$$

All linear terms drop out of the approximation since $U_{1,T}=U_{2,T}$ and $d\alpha_{1,T}=0$ in the zero-debt equal-weights cooperative equilibrium. ■

To obtain the gains from cooperation under state-contingent welfare weights, we evaluate Equation C.1 under the best response functions with and without cooperation.

Proposition 8 *The (purely) quadratic approximation to the gains from cooperation* *with state-contingent weights is given by*

$$DDU_T^{co,nc} = \frac{1+\Delta_1^{p''}}{2}\left\{\left(\frac{\chi_0}{1+\Delta_1^{p''}} - \frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\right)\bar{B}_{1,T-1} - 4\frac{1-\omega_1^c}{1+\Delta_1^{p''}}d\alpha_{1,T}\right\}^2, \quad \text{(C.3)}$$

where $DDU_T^{co,nc} = \left(U_T^{co} - \bar{U}\right) - \left(U_T^{nc} - \bar{U}^*\right)$ *is the second-order approximation of the difference in the change of (generalized) global welfare from the steady state.*

Proof. We obtain the best responses under cooperation as the first-order conditions of Equation C.1 with respect to marginal costs (in deviation from equal weights)

$$mc_{1,T} - 1 = \frac{\chi_0}{1+\Delta_1^{p''}}\left(\bar{B}_{1,T-1} - \frac{4(1-\omega_1^c)}{\chi_0}d\alpha_{1,T}\right) \quad \text{(C.4)}$$

$$mc_{2,T} - 1 = \frac{\chi_0}{1+\Delta_1^{p''}}\left(-\bar{B}_{1,T-1} + \frac{4(1-\omega_1^c)}{\chi_0}d\alpha_{1,T}\right). \quad \text{(C.5)}$$

Evaluating Equation C.1 using Equation C.4 and C.5 we obtain

$$U_T^{co} - \bar{U} \approx \frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0}{1+\Delta_1^{p''}}\bar{B}_{1,T-1} - \frac{4(1-\omega_1^c)}{1+\Delta_1^{p''}}d\alpha_{1,T}\right)^2 - \frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 + 4\chi_0\bar{B}_{1,T-1}d\alpha_{1,T}. \quad \text{(C.6)}$$

Similarly, evaluating Equation C.1 using the first-order approximation of the best-response functions absent cooperation, repeated here for convenience,

$$mc_{1,T} - 1 = -\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}} - \frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1} \quad \text{(C.7)}$$

$$mc_{2,T} - 1 = -\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}} + \frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1}, \quad \text{(C.8)}$$

delivers the following expression

$$U_T^{nc} - \bar{U}^* = -\frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\right)^2\bar{B}_{1,T-1}^2 - 2\frac{1+\Delta_1^{p''}}{2}\left(\frac{\chi_0}{1+\Delta_1^{p''}}\bar{B}_{1,T-1} - \frac{4(1-\omega_1^c)}{1+\Delta_1^{p''}}d\alpha_{1,T}\right)\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\bar{B}_{1,T-1} - \frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 + 4\chi_0\bar{B}_{1,T-1}d\alpha_{1,T} \quad \text{(C.9)}$$

where $\bar{U}^* = \bar{U} - \frac{1}{2}\left(1+\Delta_1^{p''}\right)\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)^2$. Taking the difference $DDU_T^{co,nc} = \left(U_T^{co} - \bar{U}\right) - \left(U_T^{nc} - \bar{U}^*\right)$ and rearranging terms delivers Equation C.3. ■

C.2 Gains from Cooperation Under Alternative Approaches for Selecting State-Contingent Welfare Weights

Equation C.3 approximates the gains from cooperation under any welfare weighting, $d\alpha_{1,T} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. We now describe two approaches to pin down $d\alpha_{1,T}$ as a function of $\bar{B}_{1,T-1}$, following the Pareto or Negishi approach. For both approaches, we describe how to obtain a linear relationship between $d\alpha_{1,T}$ and $\bar{B}_{1,T-1}$ and show that the gains from cooperation increase in the size of the net foreign asset position just as when we fixed the welfare weights to $\frac{1}{2}$.

C.2.1 Pareto Approach

An allocation is Pareto efficient if there is no other allocation that makes one individual better off without making another individual worse off. We implement this efficiency condition in two ways:

1. In the *asymmetric* implementation, we fix the utility level of one country (here country 1) under cooperation at the utility level the country would obtain absent cooperation — in other words, we assign all the Pareto gains to the other country. Formally, we solve the problem

$$\begin{aligned} &\max_{mc_{1,T}, mc_{2,T}} U^c_{2,T} \\ &s.t. \\ &U^c_{1,T} \geq U^{nc}_{1,T}, \end{aligned} \tag{C.10}$$

which can be recast as finding the welfare weight $d\alpha_{1,T}$ such that $U^c_{1,T} - U^{nc}_{1,T} = 0$.

2. In the *symmetric* implementation, we fix the ratio of utilities under cooperation at the same value as absent cooperation. Formally, we require

$$\frac{U^c_{1,T}}{U^c_{2,T}} = \frac{U^{nc}_{1,T}}{U^{nc}_{2,T}}, \tag{C.11}$$

which ensures that both countries will be strictly better off under cooperation than non-cooperation as long as non-cooperation is inefficient.

Proposition 9 *Under the Pareto approach for selecting state-contingent welfare weights, the linear approximation to the welfare weight on the utility of a creditor (debtor) country increases (decreases) with the size of net foreign assets:*

$$d\alpha_{1,T} = \underbrace{\left[2\frac{\omega^c_1 + \Delta^{p''}_1}{1 + \Delta^{p''}_1} + \frac{1 + \Delta^{p''}_1}{\omega^c_1 + \Delta^{p''}_1}\frac{1 - \xi^p}{\xi^p} + \frac{2 - \xi^p}{\xi^p}\right]}_{> 0 \text{ for } \xi^p \in [0,1] \text{ and } \omega^c_1 \in [\frac{1}{2}, 1]} \frac{1 + \Delta^{p''}_1}{\omega^c_1 + \Delta^{p''}_1}\frac{\chi_0 \bar{B}_{1,T-1}}{8(1 - \omega^c_1)} - h^2 \tag{C.12}$$

where

$$h := \begin{cases} 0 & \textit{if symmetric implementation,} \\ \\ \frac{1}{2}\frac{1+\Delta_1^{p''}}{\omega_1^c+\Delta_1^{p''}} & \textit{if asymmetric implementation .} \end{cases}$$

Proof. We start with the *asymmetric* implementation. To preserve the welfare ranking of allocations, we first note that a second-order approximation of the term $U_{1,T}^c - U_{1,T}^{nc}$ around the zero-debt equal-weights cooperative equilibrium is given by

$$\begin{aligned} U_{1,T} - \bar{U}_1 \approx\; & -(1-\omega_1^c)(mc_{1,T}-1) - \frac{1}{2}(\omega_1^c + \Delta_1^{p''})(mc_{1,T}-1)^2 \\ & + (1-\omega_1^c)(mc_{2,T}-1) - \frac{1}{2}(1-\omega_1^c)(mc_{2,T}-1)^2 \\ & -\chi_0 \frac{1-\xi^p}{\xi^p}(mc_{1,T}-1)\bar{B}_{1,T-1} - \chi_0\frac{1}{\xi^p}(mc_{2,T}-1)\bar{B}_{1,T-1} \\ & + 2\chi_0 \bar{B}_{1,T-1} - \frac{1}{2}\frac{\chi_0^2}{1-\omega_1^c}\bar{B}_{1,T-1}^2 \end{aligned} \tag{C.13}$$

or for the difference in utilities

$$\begin{aligned} U_{1,T}^c - U_{1,T}^{nc} \approx\; & -(1-\omega_1^c)\left(mc_{1,T}^c - mc_{1,T}^{nc}\right) - \frac{1}{2}(\omega_1^c+\Delta_1^{p''})\left[\left(mc_{1,T}^c-1\right)^2 - \left(mc_{1,T}^{nc}-1\right)^2\right] \\ & + (1-\omega_1^c)\left(mc_{2,T}^c - mc_{2,T}^{nc}\right) - \frac{1}{2}(1-\omega_1^c)\left[\left(mc_{2,T}^c-1\right)^2 - \left(mc_{2,T}^{nc}-1\right)^2\right] \\ & -\chi_0\frac{1-\xi^p}{\xi^p}\left(mc_{1,T}^c - mc_{1,T}^{nc}\right)\bar{B}_{1,T-1} - \chi_0\frac{1}{\xi^p}\left(mc_{2,T}^c - mc_{2,T}^{nc}\right)\bar{B}_{1,T-1}. \end{aligned} \tag{C.14}$$

Next we evaluate Equation C.14 using the relevant expressions for marginal costs (C.4, C.5, C.7 and C.8). In doing so, it is important to recall that for computing the second-order accurate gains from cooperation given by Equation C.3, we only need the linear relationship between $d\alpha_{1,T}$ and $\bar{B}_{1,T-1}$. Hence, we ignore the second-order terms ($\bar{B}_{1,T-1}^2$, $\bar{B}_{1,T-1}d\alpha_{1,T}$, $d\alpha_{1,T}^2$) that emerge when evaluating Equation C.14. Setting the evaluated equation equal to zero

$$\begin{aligned} 0 \;=\; & -2(1-\omega_1^c)\left\{\left(\frac{\chi_0}{1+\Delta_1^{p''}} - \frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\right)\bar{B}_{1,T-1} - 4\frac{1-\omega_1^c}{1+\Delta_1^{p''}}d\alpha_{1,T}\right\} \\ & + \frac{1+\Delta_1^{p''}}{2}\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)^2 - \left(1-2\omega_1^c-\Delta_1^{p''}\right)\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)\bar{B}_{1,T-1} \\ & -\chi_0\frac{2-\xi^p}{\xi^p}\left(\frac{1-\omega_1^c}{\omega_1^c+\Delta_1^{p''}}\right)\bar{B}_{1,T-1} \end{aligned} \tag{C.15}$$

yields the expression in Equation C.12 after rearranging terms.

Turning to the *symmetric* implementation, the condition

$$\frac{U^c_{1,T}}{U^c_{2,T}} = \frac{U^{nc}_{1,T}}{U^{nc}_{2,T}} \tag{C.16}$$

is approximated to the second order as

$$\begin{aligned} 0 \;=\; & -2(1-\omega^c_1)\left(mc^c_{1,T} - mc^{nc}_{1,T}\right) + \frac{1}{2}(1-2\omega^c_1 - \Delta^{p''}_1)\left[\left(mc^c_{1,T}-1\right)^2 - \left(mc^{nc}_{1,T}-1\right)^2\right] \\ & +2\left(1-\omega^c_1\right)\left(mc^c_{2,T} - mc^{nc}_{2,T}\right) - \frac{1}{2}\left(1-2\omega^c_1 - \Delta^{p''}_1\right)\left[\left(mc^c_{2,T}-1\right)^2 - \left(mc^{nc}_{2,T}-1\right)^2\right] \\ & -\chi_0\frac{2-\xi^p}{\xi^p}\left(mc^c_{1,T} - mc^{nc}_{1,T}\right)\bar{B}_{1,T-1} - \chi_0\frac{2-\xi^p}{\xi^p}\left(mc^c_{2,T} - mc^{nc}_{2,T}\right)\bar{B}_{1,T-1}, \end{aligned} \tag{C.17}$$

where we have made use of the fact that the approximation of the term $U^c_{2,T} - U^{nc}_{2,T}$ mirrors the one of $U^c_{1,T} - U^{nc}_{1,T}$.[41]

Evaluating Equation C.17 using the relevant expressions for marginal costs (C.4, C.5, C.7 and C.8) and ignoring the second-order terms ($\bar{B}^2_{1,T-1}$, $\bar{B}_{1,T-1}d\alpha_{1,T}$, $d\alpha^2_{1,T}$) to obtain a linear relationship between $d\alpha_{1,T}$ and $\bar{B}_{1,T-1}$ delivers

$$\begin{aligned} 0 \;=\; & -4(1-\omega^c_1)\left\{\left(\frac{\chi_0}{1+\Delta^{p''}_1} - \frac{-\chi_0\frac{1-\xi^p}{\xi^p}}{\omega^c_1+\Delta^{p''}_1}\right)\bar{B}_{1,T-1} - 4\frac{1-\omega^c_1}{1+\Delta^{p''}_1}d\alpha_{1,T}\right\} \\ & -2\left(1-2\omega^c_1-\Delta^{p''}_1\right)\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega^c_1+\Delta^{p''}_1}\left(\frac{1-\omega^c_1}{\omega^c_1+\Delta^{p''}_1}\right)\bar{B}_{1,T-1} \\ & -2\chi_0\frac{2-\xi^p}{\xi^p}\left(\frac{1-\omega^c_1}{\omega^c_1+\Delta^{p''}_1}\right)\bar{B}_{1,T-1} \end{aligned} \tag{C.18}$$

which, up to a constant, coincides with the expression in Equation C.15. ■

Making use of Equation C.12 in Equation C.3, we obtain the following proposition on the gains from cooperation under Pareto Efficiency:

Proposition 10 *The (purely) quadratic approximation to the welfare gains from cooperation under the Pareto approach for selecting welfare weights is*

$$DDU^{co,nc}_T \;=\; \frac{1+\Delta^{p''}_1}{2}\left(\frac{1}{2}\frac{1-\omega^c_1}{\omega^c_1+\Delta^{p''}_1}\right)^2\left\{\left(1+\frac{1}{1-\xi^p}\frac{\omega^c_1+\Delta^{p''}_1}{1-\omega^c_1}\right)\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega^c_1+\Delta^{p''}_1}\bar{B}_{1,T-1} - h\right\}^2. \tag{C.19}$$

where

$$h := \begin{cases} 0 & \textit{if symmetric implementation,} \\ \frac{1}{2}\frac{1+\Delta^{p''}_1}{\omega^c_1+\Delta^{p''}_1} & \textit{if asymmetric implementation}. \end{cases}$$

[41] In going from $U^c_{1,T} - U^{nc}_{1,T}$ to $U^c_{2,T} - U^{nc}_{2,T}$, the roles of mc_1 and mc_2 are switched and $\bar{B}_{1,T-1}$ is replaced by $-\bar{B}_{1,T-1}$.

Under the symmetric (asymmetric) implementation, the gains are strictly (weakly) increasing in the size of the net foreign asset position.

C.2.2 Negishi Approach

Following Negishi (1972), we set $\alpha_{1,T}$ based on the relative marginal utilities of consumption in the two countries under non-cooperative policies.

Hence, we set

$$\alpha_{1,T} = \frac{\frac{1}{\frac{\partial U_{1,T}^{nc}}{\partial C_{1,T}^{nc}}}}{\frac{1}{\frac{\partial U_{1,T}^{nc}}{\partial C_{1,T}^{nc}}} + \frac{1}{\frac{\partial U_{2,T}^{nc}}{\partial C_{2,T}^{nc}}}} = \frac{1}{1 + \frac{C_{2,T}^{nc}}{C_{1,T}^{nc}}} = \frac{1}{1 + \frac{mc_{2,T}}{mc_{1,T}} \delta_{1,T}^{2\left(1-\omega_1^c\right)}}. \tag{C.20}$$

The restatement of the Negishi weight as $\frac{1}{1+\frac{C_{2,T}^{nc}}{C_{1,T}^{nc}}}$ reflects our choice of utility functions. The subsequent restatement as $\frac{1}{1+\frac{mc_{2,T}}{mc_{1,T}}\delta_{1,T}^{2\left(1-\omega_1^c\right)}}$ reflects the relationships between consumption, marginal costs and the terms of trade, namely

$$C_{1,T} = \frac{1}{\chi_0} mc_{1,T} \left(\delta_{1,T}\right)^{-\left(1-\omega_1^c\right)}, \tag{C.21}$$

$$C_{2,T} = \frac{1}{\chi_0} mc_{2,T} \left(\delta_{1,T}\right)^{\left(1-\omega_1^c\right)}. \tag{C.22}$$

Proposition 11 *Under the Negishi approach for selecting welfare weights, the linear approximation to the welfare weight of the creditor (debtor) country increases (decreases) with the size of the net-foreign-asset position:*

$$d\alpha_{1,T} = \frac{1}{2}\chi_0 \frac{\omega_1^c + \frac{1-\xi^p}{\xi^p}\left[2(1-\omega_1^c) + \frac{1}{\theta^p}\right]}{\omega_1^c + \frac{1-\xi^p}{\xi^p}\left[1 + \frac{1}{\theta^p}\right]} \bar{B}_{1,T-1} > 0. \tag{C.23}$$

Proof. The first-order approximation to the Negishi weight is

$$\begin{aligned} d\alpha_{1,T} = \alpha_{1,T} - \frac{1}{2} &\approx -\frac{1}{4}\left[(mc_{2,T}-1) - (mc_{1,T}-1) + 2\left(1-\omega_1^c\right)\left(\delta_{1,T}-1\right)\right] \\ &= -\frac{1}{4}\left[2\frac{\chi_0 \frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} - 2\left(1-\omega_1^c\right)\left(2\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c + \Delta_1^{p''}} + \frac{\chi_0}{1-\omega_1^c}\right)\right]\bar{B}_{1,T-1} \\ &= \frac{1}{2}\chi_0 \frac{\omega_1^c + \frac{1-\xi^p}{\xi^p}\left[2(1-\omega_1^c) + \frac{1}{\theta^p}\right]}{\omega_1^c + \frac{1-\xi^p}{\xi^p}\left[1+\frac{1}{\theta^p}\right]}\bar{B}_{1,T-1} > 0. \end{aligned} \tag{C.24}$$

where the second line substitutes in the expressions for marginal costs and the terms of trade in the non-cooperative equilibrium. ■

Making use of Equation C.20 in Equation C.3, we obtain the following proposition on the gains from cooperation under the Negishi approach:

Proposition 12 *Under the Negishi approach for selecting welfare weights, the (purely)* *quadratic approximation to the welfare gains from cooperation is*

$$DDU_T^{co,nc} = \frac{1+\Delta_1^{p''}}{2}\left\{\frac{\chi_0}{1+\Delta_1^{p''}}\left(2\omega_1^c-1\right)+\frac{\chi_0\frac{1-\xi^p}{\xi^p}}{\omega_1^c+\Delta_1^{p''}}\left(1+\left(2\omega_1^c-1\right)\frac{2(1-\omega_1^c)}{1+\Delta_1^{p''}}\right)\right\}^2\bar{B}_{1,T-1}^2. \tag{C.25}$$

The gains are increasing in the size of the net-foreign-asset position.

D Appendix: Consumption Equivalent Variation

Consider the utility functions of the two countries $i = 1, 2$, repeated for convenience

$$\mathcal{U}_{i,t} = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{i,t+j} - \kappa C_{i,t+j-1}\right) - \frac{\chi_0}{1+\chi} L_{i,t+j}^{1+\chi} \right\}.$$

Define global conditional welfare to be $Welf_t = \alpha_1 \mathcal{U}_{1,t} + (1-\alpha_1)\mathcal{U}_{2,t}$. We denote by $Welf_t^{co}$ the global welfare level attained under the cooperative equilibrium, and by $Welf_t^{nc}$ the global welfare level attained under the non-cooperative equilibrium. For given paths of consumption and labor in the two countries, we are interested in sizing a permanent subsidy τ applied to the consumption utility stream of country 1 such that the level of global welfare under the non-cooperative policies is equal to the level of global welfare under the cooperative policies. Thus,

$$\begin{aligned} & \alpha_1 E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left((1+\tau) \left(C_{1,t+j}^{nc} - \kappa C_{1,t+j-1}^{nc} \right) \right) - \frac{\chi_0}{1+\chi_1} \left(L_{1,t+j}^{nc} \right)^{1+\chi} \right\} \\ + \quad & (1-\alpha_1) E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{2,t+j}^{nc} - \kappa C_{2,t+j-1}^{nc} \right) - \frac{\chi_0}{1+\chi} \left(L_{2,t+j}^{nc} \right)^{1+\chi} \right\} = Welf_t^{co}, \end{aligned}$$

which can be rewritten as

$$\frac{\alpha_1}{1-\beta} \log(1+\tau) + \alpha_1 Welf_{1,t}^{nc} + (1-\alpha_1) Welf_{2,t}^{nc} = Welf_t^{co}.$$

Rearranging terms, we obtain

$$\tau = \exp\left(\frac{1-\beta}{\alpha_1} \left(Welf_t^{co} - Welf_t^{nc} \right) \right) - 1.$$

Similarly, when considering country-specific outcomes, we compute country-specific gains for choosing nationally oriented policies over cooperation as

$$\begin{aligned} \tau_1 &= \exp\left((1-\beta) \left(Welf_{1,t}^{nc} - Welf_{1,t}^{co} \right) \right) - 1, \\ \tau_2 &= \exp\left((1-\beta) \left(Welf_{2,t}^{nc} - Welf_{2,t}^{co} \right) \right) - 1. \end{aligned}$$

Using a first-order approximation, the following relationship holds

$$-\tau = \tau_1 + \frac{1-\alpha_1}{\alpha_1} \tau_2,$$

between the global gains from cooperation τ and the country-specific gains from nationally oriented policies τ_1, τ_2.