MODELING INVESTMENT-SECTOR EFFICIENCY SHOCKS: WHEN DOES DISAGGREGATION MATTER?*

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The most straightforward way to analyze investment-sector productivity developments is to construct a two-sector model with a sector-specific productivity shock. An often used modeling shortcut accounts for such developments using a one-sector model with shocks to the efficiency of investment in a capital accumulation equation. This shortcut is theoretically justified when some stringent conditions are satisfied. Using a two-sector model, we consider the implications of relaxing several of the conditions that are at odds with the U.S. Input–Output Tables, including equal factor shares across sectors. The effects of productivity shocks to an investment-producing sector of our two-sector model differ from those of efficiency shocks to investment in a one-sector model. Notably, expansionary productivity shocks boost consumption in every period, whereas expansionary efficiency shocks cause consumption to fall substantially for many periods.

1. INTRODUCTION

In post-WWII U.S. data, the relative price of investment has a downward trend and varies over the cycle. Moreover, investment-sector productivity developments have been identified as a primary driver of the economic boom of the late 1990s.[2] Perhaps the most straightforward way to account for investment-sector productivity developments is to construct a two-sector model with investment-sector multi-factor productivity (MFP) shocks. However, the more frequently used approach relies on shocks to the marginal efficiency of investment (MEI) in a capital accumulation equation of a one sector model.[3] In fact, MEI shocks have become the leading candidate explanation for postwar business cycle fluctuations.[4] The focus of this article is on investment-sector efficiency shocks, their effects, and the implications of capturing them with MFP shocks or with MEI shocks. As we show, alternative ways of accounting for sectoral productivity developments lead to differences in both qualitative and quantitative outcomes for aggregate variables.

Greenwood et al. (1997) pioneered the MEI approach. Greenwood et al. (2000) show that their one-sector model is a special case of a model with two sectors, one that produces a good used only for equipment investment and another that produces a good used for both consumption and structures investment. Under certain conditions, an MEI shock to the equipment accumulation

equation of their one-sector model is equivalent for aggregate variables to an MFP shock to equipment production in the two-sector model. This "aggregate equivalence" result provides a basis for interpreting MEI shocks as MFP shocks.

It may come as no surprise that the conditions for aggregate equivalence are quite restrictive. Capital is taken to be perfectly mobile between sectors, and no allowance is made for costs of adjusting investment. We show that the conditions also entail a production structure that differs significantly from the one implied by the U.S. Input–Output (IO) Tables. For instance, the conditions require the same factor shares across sectors, whereas we show that factor shares are quite different across sectors. In a paper with a focus different from ours, Basu et al. (2010) also show that the structure of U.S. production implies different factor shares across sectors.[5]

We show how reasonable departures from the restrictive conditions for equivalence affect the aggregate outcomes of both sectoral MFP shocks and MEI shocks. Our model has two production sectors, a machinery-producing sector and its complement, a nonmachinery-producing sector, and is calibrated to the U.S. IO Tables and other sectoral statistics.[6] In this model, MFP increases in the machinery-producing sector have effects that are qualitatively different from MEI increases in a one-sector model, even though the models are calibrated to match the same aggregate features whenever possible. One important difference is that with MFP shocks, consumption is boosted at all horizons, whereas with MEI shocks consumption is reduced for many quarters beginning soon after the shock.[7]

Following the empirical support for the importance of investment shocks provided by Fisher (2006), Smets and Wouters (2007), and Justiniano and Primiceri (2008), a growing number of papers that estimate dynamic stochastic general equilibrium (DSGE) models have included such shocks and found them to be a major driver of business cycle fluctuations. However, these studies struggle with the problem that if MEI shocks are prominent, they may cause the unconditional correlation between investment and consumption to be counterfactually negative. MFP shocks in the machinery sector, while sharing many features with MEI shocks, can resolve this incongruence.[8]

There are other ways of generating comovement between consumption and investment. A good overview of the literature on comovement is provided by Christiano and Fitzgerald (1998).[9] The mechanism suggested by Greenwood et al. (2000) revolves around variable capacity utilization for capital. Christiano et al. (2008) point to strong consumption habits and investment adjustment costs as a mechanism for generating comovement. Eusepi and Preston (2008), Jaimovich and Rebelo (2009), Furlanetto and Seneca (2010), and Papanikolaou (2011) focus on departures from utility functions that are additively separable in consumption and leisure to generate comovement. While focusing on different issues, Schmitt-Grohe and Uribe (2011) offer yet another possible mechanism for generating comovement: correlated shocks. They notice that MFP series from aggregate growth accounting exercises and the relative price of equipment investment are cointegrated and reconcile this finding with cointegrated neutral MFP and MEI shocks.

Our approach is different in several ways. We do not have to introduce variable capacity utilization of factor inputs. The specification of preferences we consider is time separable. We obtain comovement while abstracting from consumption habits and the labor–leisure choice altogether. Although we allow for investment adjustment costs, in our two-sector model they

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[5] Their main contribution is to develop a new method of estimating sector-specific technology shocks.

[6] One of the first papers to emphasize the importance of the input–output structure for the business cycle is Long and Plosser (1983). More recent contributions include Hornstein and Praschnik (1997) and Edge et al. (2008).

[7] In related work, Swanson (2006) showed that MFP shocks at the sectoral level in a multi-sector model can lead to different aggregate implications from those of MFP shocks in a one-sector model.

[8] In Guerrieri et al. (2010) we use Monte Carlo methods to show that MFP shocks in our two-sector model are much more likely than MEI shocks in a one-sector model to produce the strong positive correlation between consumption and investment found in the data.

[9] In an open economy setting, Raffo (2010) shows that MEI shocks may help generate improvements in the terms of trade in periods of economic expansion.

are not sufficient by themselves to generate comovement. Finally, given that the conditions for aggregate equivalence fail in our two-sector model, it is possible to find that aggregate MFP measures and the relative price of equipment investment are cointegrated even when maintaining the hypothesis of orthogonal shocks across sectors.

2.   THE MODEL

Our approach to the analysis of productivity changes is a combination of the growth-accounting approach based on industrial breakdowns—in the tradition of Solow (1957) and Griliches and Jorgenson (1966) —and the approach based on final-use breakdowns typical of DSGE models.[10]

Our two-sector model has some similarities to the one posited by Greenwood et al. (2000). Both models have two production sectors and the same three final goods (equipment investment, consumption, and structures investment). However, our model emobodies three extensions. The first extension is that the outputs of both the machinery (M) and nonmachinery (N) sectors are used in " assembling" all three final goods. For example, equipment investment is assembled using machines from the M sector and distribution services from the N sector. Thus, the structure of our economy differs from that in Greenwood et al. (2000) except in the limiting case of "complete specialization in assembly" in which M output is used only in the assembly of equipment and N output is used only in the assembly of consumption and structures. In this limiting case, the machinery sector could just as well be referred to as the equipment sector, as it is in Greenwood et al. (2000).

The other two extensions are additions of two types of real rigidities. First, as has become common in DSGE models, we allow for costs of adjusting investment.[11] This extension enables us to consider conditions for equivalence under alternative specifications of these costs. Second, we allow for costs of adapting capital suitable for one sector for use in the other. This extension makes it possible for us to consider the case in which capital stocks are predetermined not only at the aggregate level but also essentially predetermined at the sectoral level.[12]

2.1. *Production Sectors.*   Our two production sectors, the M and N sectors, comprise perfectly competitive firms. Consider the representative firm in sector i (where $i \in \{M, N\}$) in period s. It hires labor (L_{is}) from households at a wage (W_s) that is same for both sectors because labor is perfectly mobile between sectors. It also rents two types of capital from households: equipment capital (K_{is}^E) and structures capital (K_{is}^S) at rentals (R_{is}^E and R_{is}^S) that are sector-specific when it is costly to reallocate capital. The firm minimizes the unit cost of producing a given number of physical units of its sector's output (Y_{is}) subject to a sector-specific Cobb–Douglas production function:

$$(1) \qquad\qquad Y_{is} = (L_{is})^{1-\alpha_i^E-\alpha_i^S} \left(K_{is}^E\right)^{\alpha_i^E} \left(K_{is}^S\right)^{\alpha_i^S}.$$

The factor shares for the two types of capital are α_i^E and α_i^S.

There is a multifactor productivity (MFP) process A_s that determines the efficiency units generated by physical machinery output ($Y_{Ms}^A = A_s Y_{Ms}$).[13] For example, for computers, Y_{Ms} can be thought as the number of computers produced and Y_{Ms}^A as the computing power generated

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[10] We refer to " production sectors" rather than "industries" because the former terminology is more common in the DSGE literature to which our article is somewhat more closely related.

[11] Investment adjustment costs are not a part of the model developed by Greenwood et al. (1997) and Greenwood et al. (2000), but are a common ingredient of models developed subsequently that also incorporate MEI shocks.

[12] Ramey and Shapiro (1998) document the high costs of reallocating capital across sectors. Greenwood et al. (1997) abstract from adjustment costs of capital altogether. Greenwood et al. (2000) include sector-specific costs of adjusting capital over time, but continue to allow for the costless reallocation of capital across sectors within a period.

[13] Note that we do not consider MFP shocks to the non-machinery sector.

by these computers. We find it convenient to account separately for physical and efficiency units when comparing sectoral MFP shocks with MEI shocks.

Since it is competitive and there are constant returns to scale, the firm ends up selling at a price equal to unit cost. Let P_{is} represent the factor cost of a unit of physical output i.[14] We assume that the N good is the numeraire, so $P_{Ns} = 1$. The factor cost of a physical unit of machinery is P_{Ms} and the cost of an efficiency unit of machinery is $P_{Ms}^A = \frac{P_{Ms}}{A_s}$ so that

$$(2) \qquad P_{Ms} Y_{Ms} = \left(\frac{P_{Ms}}{A_s} \right) A_s Y_{Ms} = P_{Ms}^A Y_{Ms}^A.$$

2.2. *Final Goods.*   There are three final goods: a consumption good (C_s) and two investment goods, one (J_s^E) used for gross investment in E capital stocks and the other (J_s^S) used for gross investment in S capital stocks. These goods are assembled by perfectly competitive final goods firms that use as inputs the outputs of the two production sectors, and these final goods are measured in efficiency units. When we find it expedient for the exposition, we use an upper bar to denote final goods measured in physical units.

The assembly function for C_s is a constant elasticity of substitution (CES) function of the two consumption inputs, efficiency units of M goods ($A_s C_{Ms}$) along with N goods (C_{Ns}):

$$(3) \qquad C_s = \left[\phi_M^C \left(\frac{A_s C_{Ms}}{\phi_M^C} \right)^{\frac{\sigma_C-1}{\sigma_C}} + \phi_N^C \left(\frac{C_{Ns}}{\phi_N^C} \right)^{\frac{\sigma_C-1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C-1}},$$

where ϕ_M^C and ϕ_N^C are the weights for M and N goods, and σ_C is the elasticity of substitution between M and N goods in the assembly of C_s.

The assembly functions for J_s^E and J_s^S are CES functions of the two investment inputs, efficiency units of M goods ($A_s I_{Ms}^E$, $A_s I_{Ms}^S$) along with N goods (I_{Ns}^E, I_{Ns}^S):

$$(4) \qquad J_s^E = \left[\phi_M^E \left(\frac{A_s I_{Ms}^E}{\phi_M^E} \right)^{\frac{\sigma_E-1}{\sigma_E}} + \phi_N^E \left(\frac{I_{Ns}^E}{\phi_N^E} \right)^{\frac{\sigma_E-1}{\sigma_E}} \right]^{\frac{\sigma_E}{\sigma_E-1}},$$

$$(5) \qquad J_s^S = \left[\phi_M^S \left(\frac{A_s I_{Ms}^S}{\phi_M^S} \right)^{\frac{\sigma_S-1}{\sigma_S}} + \phi_N^S \left(\frac{I_{Ns}^S}{\phi_N^S} \right)^{\frac{\sigma_S-1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S-1}},$$

where ϕ_M^E, ϕ_N^E, ϕ_M^S, and ϕ_N^S are the weights given to M and N goods, and σ_S and σ_E are the elasticities of substitution between M and N goods.

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[14] For example, P_M is the multiplier in the Lagrangian expression ($\mathcal{L}_M$) used to the minimize costs of producing a given physical quantity Y_M:

$$\mathcal{L}_M = WN_M + R_M^E K_M^E + R_M^S K_M^S + P_M \left\{ Y_M - (L_M)^{1-\alpha_N^E-\alpha_N^S} \left(K_M^E \right)^{\alpha_N^E} \left(K_M^S \right)^{\alpha_N^S} \right\},$$

where time subscripts have been omitted for simplicity.

The assembly firms minimize the unit cost of producing efficiency units of consumption, equipment, and structures.[15] Because they are perfectly competitive, firms end up selling final goods at prices that are equal to these costs and that are indicated by P_s^C, $P_s^{J^E}$, and $P_s^{J^S}$. We assume that the assembly functions for both C_s and J_s^S are N-intensive relative to the function for J_s^E.

There is a shock Z_s to the marginal efficiency of investment (MEI) that further enhances the efficiency of J_s^E, the efficiency unit of equipment assembled using M and N inputs. The final total amount of equipment efficiency units is given by $Z_s J_s^E$ and the all-in unit cost is $\frac{P_s^{J^E}}{Z_s}$ so that

$$(6) \qquad P_s^{J^E} J_s^E = \left(\frac{P_s^{J^E}}{Z_s}\right) Z_s J_s^E.$$

For example, the expressions $Z_s J_s^E$ and $\left(\frac{P_s^{J^E}}{Z_s}\right)$ in the model are analogous to the measures of computer output and the price of computer output in the NIPA.

We sometimes refer to the case of "partial specialization" in assembly. Under partial specialization, the assembly functions for C and J^S depend only on the N good:

$$(7) \qquad C_s = C_{Ns}, \qquad J_s^S = I_{Ns}^S, \qquad Y_{Ns} = C_{Ns} + I_{Ns}^S + I_{Ns}^E,$$

and the assembly function for total efficiency units of equipment investment, $Z_s J_s^E$, is Cobb–Douglas:

$$Z_s J_s^E = Z_s \left(A_s I_{Ms}^E\right)^{\phi_M^E} \left(I_{Ns}^E\right)^{\phi_N^E} = Z_s \left(A_s\right)^{\phi_M^E} \left(I_{Ms}^E\right)^{\phi_M^E} \left(I_{Ns}^E\right)^{\phi_N^E} = Z_s A_s^{\phi_M^E} \bar{J}_s^E,$$

$$(8) \qquad \bar{J}_s^E = \left(I_{Ms}^E\right)^{\phi_M^E} \left(I_{Ns}^E\right)^{\phi_N^E},$$

where $Z_s J_s^E$ incorporates the enhancements coming from Z as well as from A and where $\bar{J}_s^E$ represents equipment investment in "physical units" (number of computers). Therefore,

$$(9) \qquad \left(\frac{P_s^{J^E}}{Z_s}\right) Z_s J_s^E = \left(\frac{P_s^{\bar{J}^E}}{Z_s \left(A_s\right)^{\phi_M^E}}\right) Z_s A_s^{\phi_M^E} \bar{J}_s^E = P_s^{\bar{J}^E} \bar{J}_s^E,$$

where $P_s^{J^E}$ is the cost of a unit of J_s^E and $P_s^{\bar{J}^E}$ is the cost of a unit of $\bar{J}_s^E$.

Partial specialization has the case of "complete specialization" as a limit. Under complete specialization, the assembly function for equipment investment depends only on the M good ($\phi_M^E = 1$, $\phi_N^E = 0$) and all machinery output is used for equipment investment so that

$$(10) \qquad Z_s J_s^E = Z_s A_s I_{Ms}^E = Z_s A_s Y_{Ms}, \qquad C_s + J_s^S = C_{Ns} + I_{Ns}^S = Y_{Ns}.$$

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[15] For example, P^{J^E} is the multiplier in the Lagrangian expression ($\mathcal{L}_{J^E}$) used to the minimize costs of producing a given quantity J^E:

$$\mathcal{L}_{J^E} = P_M I_M^E + I_N^E + P^{J^E} \left\{ J^E - \left[\phi_M^E \left(\frac{A_s I_M^E}{\phi_M^E}\right)^{\frac{\sigma_E - 1}{\sigma_E}} + \phi_N^E \left(\frac{I_N^E}{\phi_N^E}\right)^{\frac{\sigma_E - 1}{\sigma_E}} \right]^{\frac{\sigma_E}{\sigma_E - 1}} \right\},$$

where time subscripts have been omitted for simplicity.

The complete specialization case is important because, beginning with Greenwood et al. (1997), the literature that relates MEI shocks to MFP shocks focuses almost exclusively on this case. For this reason, we assume complete specialization ($\phi_M^E = 1$) in our baseline case.

2.3. *Tastes and Constraints.*   In period t, the representative household supplies a fixed amount of labor L and maximizes the following intertemporal utility function:[16]

$$(11) \qquad \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma} - 1}{1 - \gamma}.$$

The household also chooses holdings of a single bond (B_s) denominated in the N good (the numeraire good for the model). In addition, for each of the four inherited capital stocks ($D_{Ms}^E, D_{Ns}^E, D_{Ms}^S$, and D_{Ns}^S), the household decides how much to adapt to obtain the four capital stocks rented out for use in production ($K_{Ms}^E, K_{Ns}^E, K_{Ms}^S$, and K_{Ns}^S) as well as the fractions ($j_{Ms}^E, j_{Ns}^E, j_{Ms}^S$, and j_{Ns}^S) of investment of the two types (J_s^E or J_s^S) to be added to the four capital stocks. The distinction between capital inherited from the previous period, the D_{is}^j stocks, and capital used in production, the K_{is}^j stocks, allows us to nest in the same model the case in which capital is predetermined only at the *aggregate* level and the case in which capital is essentially predetermined also at the *sectoral* level.

The household is subject to period budget constraints. In each period, factor income plus income from bonds held in the previous period must be at least enough to cover purchases of final goods (consumption goods and the two types of investment goods) as well as bonds:

$$(12) \qquad \begin{aligned} W_s L &+ R_{Ms}^E K_{Ms}^E + R_{Ms}^S K_{Ms}^S + R_{Ns}^E K_{Ns}^E + R_{Ns}^S K_{Ns}^S + \rho_{s-1} B_{s-1} \\ &= P_s^C C_s + P_s^{J^E} J_s^E + P_s^{J^S} J_s^S + B_s, \end{aligned}$$

where $R_{Ms}^E, R_{Ms}^S, R_{Ns}^E, R_{Ns}^S$ are the rental rates for the capital stocks used in production. The term ρ_{s-1} is the gross return on bonds.

The household is subject to technological constraints when allocating capital. It inherits four capital stocks from the previous period. Inherited capital suited for one sector can be adapted for use in the other sector before being rented out, but only by incurring increasing marginal costs. For example, inherited equipment capital (D_{Ms}^E) suited for the M sector can be adapted for use in the N sector (K_{Ns}^E). Therefore, the capital of type h actually available for production in sector i in period s depends on how much has been adapted for production in that sector:

$$(13) \qquad \begin{aligned} K_{Ms}^h + K_{Ns}^h &= D_{Ms}^h \left[1 - \frac{\omega^h}{2} \left(\frac{K_{Ms}^h}{D_{Ms}^h} - 1 \right)^2 \right] \\ &+ D_{Ns}^h \left[1 - \frac{\omega^h}{2} \left(\frac{K_{Ns}^h}{D_{Ns}^h} - 1 \right)^2 \right], \quad h \in \{E, S\}. \end{aligned}$$

Here, we restrict our attention to two special cases: the case in which capital can be adapted at no cost ($\omega^h = 0$) so that capital is predetermined only at the aggregate level and the case in which the marginal cost of adapting capital becomes prohibitive ($\omega^h \to \infty$) so that capital is predetermined at the sectoral level as well.

The household is also subject to technological constraints when accumulating capital. The accumulation equations for structures capital are more straightforward, so we consider them

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[16] The assumptions of fixed aggregate labor supply and perfect mobility of labor across sectors were made for simplicity, given our already involved structure with many sectors. Relaxing either of these assumptions matters for the issue of comovement. Katayama and Kim (2012) relax both assumptions.

first. Let D_{is}^S represent the amount of S capital available for production in sector i in period s without incurring any costs of adaptation:

$$(14) \qquad D_{is}^S = \left(1 - \delta_i^S\right) K_{is-1}^S + j_{is-1}^S J_{s-1}^S - \frac{v_{0i}^S}{2} j_{is-1}^S J_{s-1}^S \left(\frac{j_{is-1}^S J_{s-1}^S}{j_{is-2}^S J_{s-2}^S} - 1\right)^2, \quad i \in \{M, N\},$$

where j_{is-1}^S is the proportion of total structures investment in period $s-1$ that is added to the structures capital suitable for sector i in that period. D_{is}^S has three components represented by the three terms on the right-hand side of Equation (14). The first is the amount of S capital actually used in production in sector i in period $s-1$ remaining after depreciation. The second is the amount of S investment added to structures capital suitable for sector i in period $s-1$. The third represents the adjustment costs incurred if the S investment in a given type of capital in period $s-1$ differs from that in period $s-2$. It is important to note that although the MEI shock Z_s does not enter the accumulation equations for structures capital by assumption, the MFP shock A_s does enter through J_s^S except in the case of complete specialization in assembly in which $J_s^S = I_{Ns}^S$.

The accumulation equations for equipment capital are less straightforward because of the distinction between physical units and efficiency units. Let D_{is}^E represent the amount of E capital available for production in sector i in period s without incurring any costs of adaptation:

$$(15) \qquad D_{is}^E = \left(1 - \delta_i^E\right) K_{is-1}^E + Z_{s-1} j_{is-1}^E J_{s-1}^E$$
$$+ \frac{v_{0i}^E}{2} \left(Z_{s-1}\right)^{v_1^E} j_{is-1}^E J_{s-1}^E \left[\left(\frac{Z_{s-1}}{Z_{s-2}}\right)^{v_2^E} \frac{j_{is-1}^E J_{s-1}^E}{j_{is-2}^E J_{s-2}^E} - 1\right]^2, \quad i \in \{M, N\},$$

where j_{is-1}^E is the proportion of total equipment investment that is devoted to accumulation of structures capital suited for sector i in period $s-1$, and where the parameters v_1^E and v_2^E can take on the values of one or zero.[17] Like D_{is}^S, D_{is}^E has three components. The first components of D_{is}^S and D_{is}^E are completely analogous. The second component of D_{is}^E is the amount of investment in equipment capital suited for sector i measured in efficiency units. It reflects the increase in the efficiency of the machinery input resulting from the MFP shock A_s, which is imbedded in J_s^E and the increase in efficiency resulting from the MEI shock Z_s. The third component represents investment adjustment costs. If $v_1^E = v_2^E = 1$, then adjustment costs apply to efficiency units no matter whether A_s or Z_s is the source of increased efficiency. We consider below the implication of zero values for both v_1^E and v_2^E or v_2^E alone.

It is instructive to consider the case of Cobb–Douglas assembly for equipment in which D_{is}^E is given by

$$(16) \qquad D_{is}^E = \left(1 - \delta_i^E\right) K_{is-1}^E + Z_{s-1} \left(A_{s-1}\right)^{\phi_M^E} j_{is-1}^E \bar{J}_{s-1}^E - \frac{v_{0i}^E}{2} \left(Z_{s-1}\right)^{v_1^E} \left(A_{s-1}\right)^{\phi_M^E v_1^E} j_{is-1}^E \bar{J}_{s-1}^E$$
$$\times \left[\left(\frac{Z_{s-1}}{Z_{s-2}}\right)^{v_2^E} \left(\frac{A_{s-1}}{A_{s-2}}\right)^{\phi_M^E v_1^E} \frac{j_{is-1}^E \bar{J}_{s-1}^E}{j_{is-2}^E \bar{J}_{s-2}^E} - 1\right]^2, \quad i \in \{M, N\}.$$

The first component of D_{is}^E is the same as in the general case. The second component, investment in sector i measured in efficiency units (MEI enhanced computing power), can be expressed as the product of two terms, an efficiency enhancement term $Z_s(A_s)^{\phi_M^E}$ and investment

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[17] For simplicity we assume that depreciation rates (δ_i^E and δ_i^S) and investment adjustment-cost parameters (v_{0i}^E and v_{0i}^S) may differ between types of capital but are the same across sectors of use.

TABLE 1
AGGREGATION EQUATIONS

$\bar{J}^E_s = \bar{J}^E_{Ms} + \bar{J}^E_{Ns}$	$J^S_s = J^S_{Ms} + J^S_{Ns}$
$K^E_s = K^E_{Ms} + K^E_{Ns}$	$K^S_s = K^S_{Ms} + K^S_{Ns}$
$Y_s = Y_{Ms} + Y_{Ns}$	

measured in "physical units" $j^E_{is-1}\bar{J}^E_{s-1}$ (where $\bar{J}^E$ is defined in Equation (8)). For the third component, investment adjustment costs, there are two versions that are consistent in the sense that whenever A and Z appear in the accumulation equations, they appear together in the same function $(Z_s(A_s)^{\phi^E_M})$. First, if $v^E_1 = v^E_2 = 1$, then adjustment costs depend on efficiency units. Second, if $v^E_1 = v^E_2 = 0$, then adjustment costs depend on physical units. In a third version where $v^E_1 = 1$ but $v^E_2 = 0$, the two efficiency factors A and Z do not always appear together in the same function. This last version is of interest because papers that attempt to capture the importance of MEI shocks for the business cycle routinely incorporate investment adjustment costs that include some efficiency enhancements but not others.[18] At least to us, it is not obvious how investment adjustment costs should be modeled.

The final household constraint is that for each type of investment good the proportions of the total amount added to the two capital stocks of the same type must sum to one:

$$1 = j^E_{Ms} + j^E_{Ns}, \quad 1 = j^S_{Ms} + j^S_{Ns}.$$

2.4. *Market Clearing.* Market clearing requires that the outputs of the production sectors must be used up in the assembly of final goods:

$$Y_{Ms} = C_{Ms} + I^E_{Ms} + I^S_{Ms}, \quad Y_{Ns} = C_{Ns} + I^E_{Ns} + I^S_{Ns},$$

that labor demand equal labor supply,

(17) $$L_{Ms} + L_{Ns} = L,$$

and that the bond be in zero net supply

(18) $$B_s = 0.$$

The conditions that firms' demands for K^E_{Ms}, K^E_{Ns}, K^S_{Ms}, and K^S_{Ns} equal households' supplies are imposed implicitly by using the same symbol for both.

3. EQUIVALENCE

This section sets out conditions under which the aggregate effects of an MFP shock in the machinery sector of our model can be reproduced by an MEI shock to equipment investment. First, we state necessary and sufficient conditions for the effects to be equivalent in a two-sector model, that is, two-sector equivalence (2SE). Second, we specify additional conditions under which these effects can be captured by the one-sector model as described in Tables 1 and 2. In this sense, there is aggregate equivalence (AE).[19] Support for our assertions can be

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[18] For example, both Smets and Wouters (2007) and Christiano et al. (2007) used the third version.
[19] Greenwood et al. (1997, 2000) state sufficient conditions for AE in the case with Cobb–Douglas production functions, complete specialization in assembly, and no adjustment costs for investment. Oulton (2007) extends the preceding analysis to the case with general constant returns to scale (CRTS) production functions; Greenwood and Krusell (2007) provide further discussion.

TABLE 2

ONE-SECTOR MODEL UNDER ASSUMPTIONS FOR AGGREGATE EQUIVALENCE

Utility Maximization Problem of Households

$\max_{C_s, J_s^E, J_s^S, K_s^E, K_s^S, B_s} \mathcal{L} = \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}-1}{1-\gamma}$
subject to the constraints:

$K_s^E = (1-\delta^E)K_{s-1}^E + Z_{s-1}A_{s-1}^{\phi_M^E}\bar{J}_{s-1}^E - \frac{v_0^E}{2}Z_{s-1}A_{s-1}^{\phi_M^E}\bar{J}_{s-1}^E \left[\left(\frac{Z_{s-1}}{Z_{s-2}}\right)\left(\frac{A_{s-1}}{A_{s-2}}\right)^{\phi_M^E} \frac{\bar{J}_{s-1}^E}{\bar{J}_{s-2}^E} - 1 \right]^2$

$K_s^S = (1-\delta^S)K_{s-1}^S + J_{s-1}^S - \frac{v_0^S}{2}J_{s-1}^S \left(\frac{J_{s-1}^S}{J_{s-2}^S} - 1\right)^2$

$W_s\bar{L} + R_s^E K_s^E + R_s^S K_s^S + \rho_{s-1}B_{s-1} = C_s + \bar{J}_s^E + J_s^S + B_s$

Cost Minimization Problem of Firms

$\min_{K_s^E, K_s^S} W_s\bar{L} + R_s^E K_s^E + R_s^S K_s^S$
subject to the constraint:
$Y_s = (L_s)^{1-\alpha^E-\alpha^S}(K_s^E)^{\alpha^E}(K_s^S)^{\alpha S}$

Equilibrium Conditions

$Y_s = C_s + \bar{J}_s^E + J_s^S$
$B_s = 0$

NOTES: All markets are assumed to be competitive. Recall that $\bar{J}_s^E$ represents equipment investment in physical units. We left both shocks Z_s and A_s in the description of the model to underscore their equivalence for aggregate variables.

found in Section 5.1. This section also contains simulation results for a calibration that includes adjustment costs for investment and satisfies the conditions for AE. We use these results as a benchmark against which to compare results for calibrations that do not satisfy the conditions.

3.1. *Conditions.* First, we state a set of conditions (set A) that are necessary and sufficient for two-sector equivalence (2SE). By 2SE we mean that MFP shocks and MEI shocks are equivalent in our two-sector model with distinct production functions in the two sectors. In particular, an MFP shock (A) that raises output in the M sector by a given percentage has the same sectoral and aggregate effects as a pair of MEI shocks (Z) that push up the effectiveness of equipment investment in both sectors by that given percentage. The set A conditions are[20]

A-1. Assembly of both consumption and structures investment is specialized in nonmachinery output ($C_s = C_{Ns}, J_s = J_{Ns}$).[21]

A-2. Assembly of equipment investment is a Cobb–Douglas function of machinery and non-machinery outputs (with an important limiting case in which it is specialized in machinery output).

A-3. If there are adjustment costs for equipment investment, MFP shocks and MEI shocks enter the costs combined in the same function wherever they appear ($v_1^E = v_2^E = 0$ or $v_1^E = v_2^E = 1$).

Previous discussions of equivalence assume that (using our terminology) assembly is completely specialized, and investment adjustment is costless.[22] Under these assumptions, our conditions for 2SE are met, but the assumptions are unnecessarily restrictive.

We extend the conditions for equivalence in two ways. First, we show that specialization in assembly of consumption and structures is necessary for equivalence but specialization of

[20] Throughout our discussion we maintain two standard assumptions. Production functions exhibit constant returns to scale, and adjustment costs are homogeneous of degree zero in current and lagged investment.

[21] Even though it is standard to assume specialization in assembly in DSGE models, in fact the outputs of several sectors are often used in the assembly goods for final uses. In particular, the final-use equipment investment as it appears in the NIPA is a combination of machinery with transportation and distribution services.

[22] Greenwood et al. (1997) and Oulton (2007) assume that (using our terminology) assembly is completely specialized and that investment adjustment is costless.

assembly of equipment is not. (Conditions A-1 and A-2 are the conditions for partial special-ization.) Second, we identify conditions under which there is equivalence when there are costs of adjusting investment.

Suppose set A conditions are fulfilled. Adding the conditions in set B yields a set of conditions that are sufficient for AE. By AE we mean that the two-sector model of the text with its particular functional forms and 2SE reduces to the one-sector model in Table 1 with its particular functional forms. AE implies that MFP shocks in the machinery sector of our two-sector model have the same effects on aggregate variables (including those defined in Table 1) as appropriately scaled MEI shocks in the equipment accumulation equation of the one-sector model.

The set B conditions are

B-1. The production functions for M and N are identical up to a multiplicative factor ($\alpha_M^E = \alpha_N^E = \alpha^E$, $\alpha_M^S = \alpha_N^S = \alpha^S$).[23]

B-2. Capital is perfectly mobile; that is, inherited stocks of both equipment and structures capital can be rented to either production sector in the current period without incurring adaptation costs no matter which sector they were rented to in the previous period.[24]

B-3. Depreciation rates for a given type of capital (equipment or structures) are identical for the M and N sectors ($\delta_M^S = \delta_N^S = \delta^S$, $\delta_M^E = \delta_N^E = \delta^E$).

B-4. Adjustment costs for a given type of investment (equipment or structures) are given by the same functional form ($v_{0M}^S = v_{0N}^S = v_0^S$, $v_{0M}^E = v_{0N}^E = v_0^E$). For each stock of capital the share of a given type of investment it receives is constant over time ($j_{Ms-2}^E = j_{Ms-1}^E = j_M^E$, which implies $j_{Ns-2}^E = j_{Ns-1}^E = j_N^E$).

If the conditions in both set A and set B are met, then there is AE whether or not investment adjustment costs are present.[25] We have not found other statements of sufficient conditions for AE when investment adjustment costs are present, our B-4.

We can draw conclusions about the necessity of some of the conditions in Set B. For our model to be reduced to the equations in Tables 1 and 2, B-3 and B-4 are necessary, but B-2 is not. We can show that B-1 is necessary in the complete-specialization case with one kind of capital when both factors of production are mobile. To see that B-2 is not necessary consider an economy in which one of the two capital stocks is completely immobile; in that economy, the production functions can be represented by the one-sector model in Table 2.[26]

3.2. *Baseline Calibration Ensuring Equivalence.* Table 3 summarizes the parameter choices for a baseline calibration that we use to illustrate aggregate equivalence (AE) between MEI and MFP shocks under our extended conditions. To facilitate comparisons with previous work on MEI shocks, we adhere to the parameter choices of Greenwood et al. (1997) whenever possible.[27] Accordingly, the output share of equipment in both the M and N sectors is 17% and the share of structures is 13%. The parameters governing the assembly functions are set so that there is complete specialization: consumption and structures investment are assembled using inputs from the N sector only, whereas equipment investment is assembled using inputs from the M sector only.[28] The depreciation rates for equipment and structures capital are 3.1% per quarter and 1.4% per quarter, respectively. The discount factor is set at 0.99, consistent with an

[23] We have assumed that the production functions for machinery and non-machinery are Cobb–Douglas. As shown by Oulton (2007), we could have used any constant-returns-to-scale production function.

[24] Instead of assuming that capital is perfectly mobile, Greenwood et al. (2000) assume that firms themselves can move between sectors at will.

[25] Greenwood et al. (2000) and Oulton (2007) have shown that in the absence of investment adjustment costs, conditions B-1 to B-3 are sufficient for aggregate equivalence.

[26] We conjecture that, when there are two or more factors of production, it is sufficient for all but one factor to be mobile. For example, see Sargent (1987) for a case of an immobile capital stock when one good is produced by multiple firms.

[27] For simplicity, we abstract from trend growth as well as capital and labor taxes, whereas Greenwood et al. (1997) incorporate them in their model.

[28] The substitution elasticities between inputs in assembly become irrelevant under complete specialization.

TABLE 3
MODEL CALIBRATION FOR BASELINE EXPERIMENT

Parameter	Determines	Parameter	Determines
	Utility Function		
$\gamma = 1$	Intertemporal consumption elast. $= 1/\gamma$	$\beta = 0.99$	Discount factor
	Depreciation Rates		
$\delta^E = 0.031$	Equipment capital	$\delta^S = 0.014$	Structures capital
	Adaptation Costs		
$\omega^E = 0$	M, N Equipment Capital	$\omega^S = 0$	M, N Structures Capital
	Adjustment Costs		
$\nu_0^E = 4$ $\nu_1^E = \nu_2^E = 1$	M, N Equipment Investment M, N Equipment Investment	$\nu_0^S = 4$	M,N Structures Investment
	M Goods Production		
$\alpha_M^N = 0.7$ $\alpha_M^S = 0.13$	Labor share Structures share	$\alpha_M^E = 0.17$	Equipment share
	N Goods Production		
$\alpha_N^N = 0.7$ $\alpha_N^S = 0.13$	Labor share Structures share	$\alpha_N^E = 0.17$	Equipment share
	Consumption Assembly		
$\phi_M^C = 0$	M goods intensity	$\phi_N^C = 1$	N goods intensity
	Assembly of Equipment Investment		
$\phi_M^E = 1$	M goods intensity	$\phi_N^E = 0$	N goods intensity
	Assembly of Structures Investment		
$\phi_M^S = 0$	M goods intensity	$\phi_N^S = 1$	N goods intensity

annualized real interest rate of 4%. The intertemporal substitution elasticity for consumption is taken to be 1.

Our baseline calibration includes one major departure from Greenwood et al. (1997): There are adjustment costs for investment in accord with recent common practice. The parameters governing adjustment costs for both types of investment (ν_{0i}^S and ν_{0i}^E) are set to 4. Adjustment costs are assumed to depend on efficiency units ($\nu_1^E = \nu_2^E = 1$).

In what follows, we present several figures. In all of them, the sizes of the shocks are normalized so that aggregate output (in quality-adjusted units at constant prices) increases by 1% in the long run.[29] The results in the figures are obtained from a standard first-order perturbation solution, as implemented in the Dynare suite of programs.

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[29] In multisector models there are several ways of aggregating sectoral outputs depending, for instance, on which good is chosen as the numeraire. We focus on a measure of aggregate output that sums sectoral outputs at constant prices after adjusting for quality. This measure is defined as $Y_{CPs} = C_{Ms} + C_{Ns} + Z_{Ms}^E A_s \bar{J}_{Ms}^E + Z_{Ns}^E J_{Ns}^E + Z_{Ms}^S A_s \bar{J}_{Ms}^S + Z_{Ns}^S A_s \bar{J}_{Ns}^S$. This approach can be shown to be first-order equivalent to a Tornqvist, chain-weighted index.
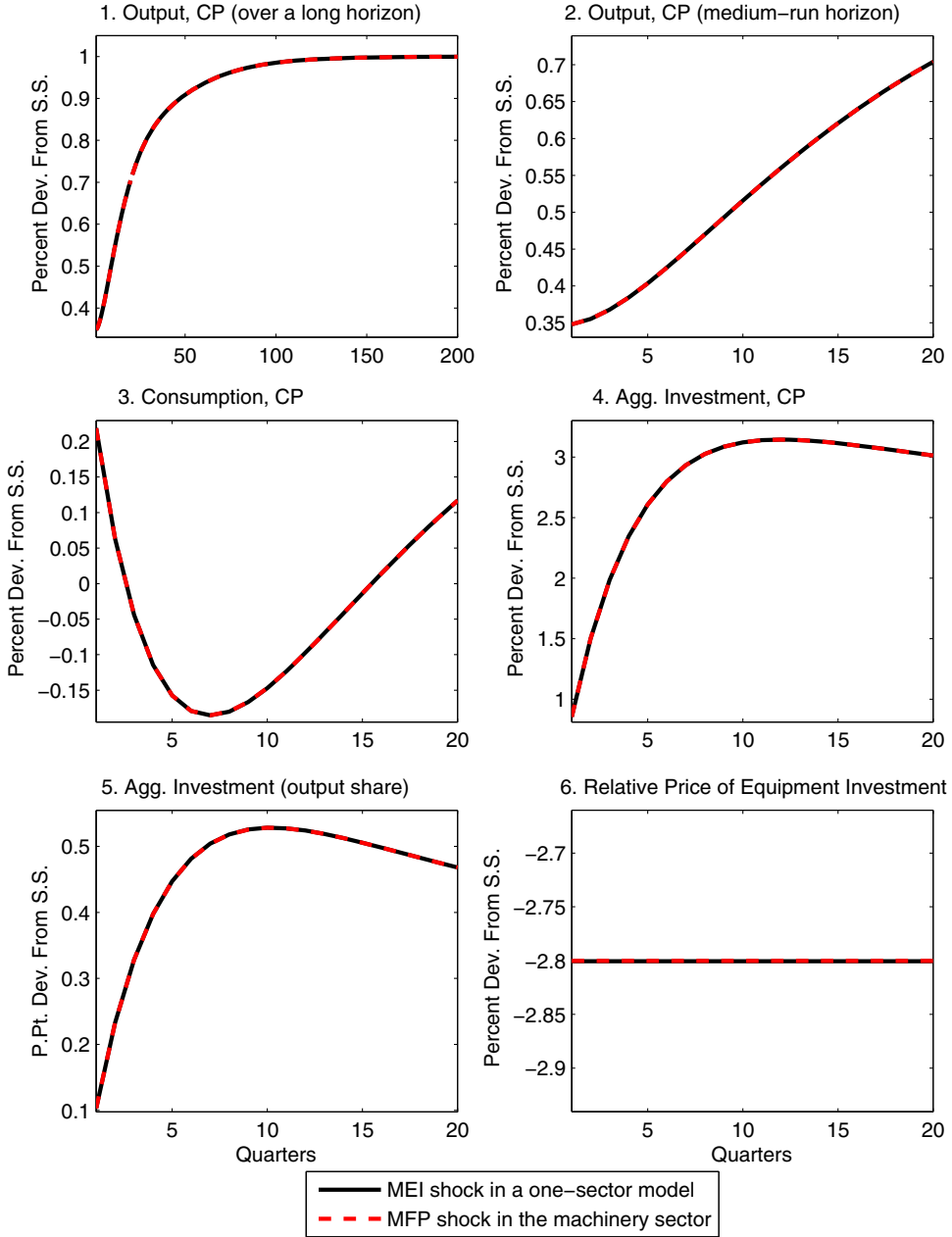
FIGURE 1

EQUIVALENT MEI AND SECTORAL MFP SHOCKS UNDER BASELINE CALIBRATION

3.3. *A Numerical Illustration.* Figures 1 and 2 show the effects of two distinct shocks with the baseline calibration. The dashed lines represent the effects of a permanent MFP shock in the *M* sector. The solid lines relate to a permanent shock to Z_s, the level of investment-specific technology. In this case, we could have cut off the model's sectoral details following Greenwood et al. (1997) and have simply obtained the aggregate responses from a canonical one-sector RBC model augmented with an MEI shock in the capital accumulation equation as described in Table 2.
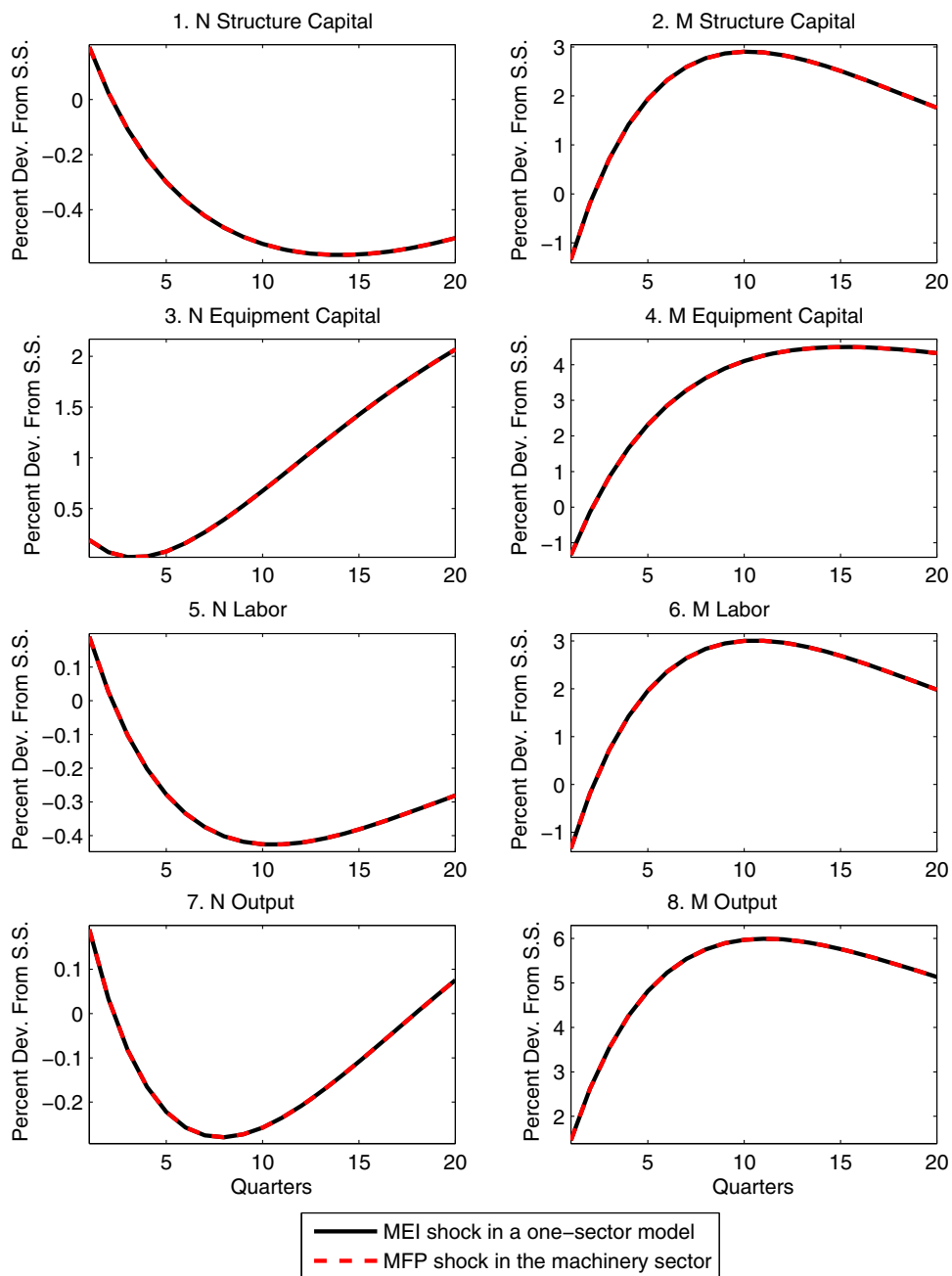
FIGURE 2

EQUIVALENT MEI AND SECTORAL MFP SHOCKS UNDER BASELINE CALIBRATION (SECTORAL DETAILS)

As intended, with the baseline calibration, these shocks produce equal effects on the aggregate variables as shown in Figure 1 since the requirements for AE (discussed above) between MFP shocks in the machinery sector and MEI shocks to equipment accumulation are satisfied. For this calibration, the (quality-adjusted) relative price of equipment investment $\left(\frac{P_s^{IE}}{Z_s} \right)$ mirrors the path of the shocks, as shown in the bottom right panel of Figure 1.

The capital accumulation process adds persistence to the effects of the shocks so that output takes a considerable number of quarters to approach its new steady-state level. The top two panels in the figure show the output response but focus on different horizons so as to depict both the medium- and long-run effects.

Both shocks make it possible to produce efficiency units of equipment investment with smaller amounts of factor inputs, regardless of which sector receives the investment. Taking account of investment adjustment costs has significant implications. Were it not for these costs, the substitution effect associated with the shocks would be so strong as to cause an immediate buildup of the equipment and structures capital stocks in the M sector. Recall that under the conditions for aggregate equivalence capital is costlessly adaptable for use in different sectors. Therefore, labor and both kinds of capital inputs would be transferred immediately away from the N sector and into the M sector. Without investment adjustment costs, consumption would drop on impact and then increase as higher production in the M sector would push up the equipment capital stock in the N sector. However, with quadratic adjustment costs in investment, it becomes costly to ramp up equipment investment, reducing the incentive to transfer factor inputs across sectors. Instead of spiking up, aggregate investment follows a hump shape. Accordingly, consumption rises on impact and then drops below zero for a protracted number of periods.

The consumption share of output takes a long time to recover as implied by Figure 1 (the consumption share is the mirror image of the investment share). As shown in Figure 2, according to the baseline calibration, N-sector goods are the sole input in the assembly of consumption. For many quarters beginning soon after the shock, N-sector output goes down, as labor and structure capital are moved to the sector that received the shock. Then, part of N-sector output is devoted to pushing up the N sector's stock of structures.

4. DEPARTURES FROM AGGREGATE EQUIVALENCE

The simulations in Figures 3 and 4 illustrate that the effects of an MFP shock in the machinery sector of a two-sector model and those of an MEI shock in a one-sector model can differ substantially when there are departures from the conditions for aggregate equivalence summarized in Section 3.1. For example, departures from the baseline calibration can generate qualitative differences between the consumption responses to MFP and MEI shocks. Although the response to the MFP shock is such that consumption never falls, in response to the MEI shock consumption falls for many quarters beginning soon after the shock.

4.1. *Adjustment Costs and Aggregate Equivalence.* The first comparison shown in Figure 3 involves the solid and dotted lines. As in Figure 1, the solid lines show the effects on aggregate variables of a machinery-sector MFP (or an MEI) shock when the conditions for aggregate equivalence are met. One of the conditions for aggregate equivalence in Section 3.1 is that adjustment costs depend on either efficiency units only (as is the case with the solid line) or physical units only.

The dotted lines show the effects of a machinery-sector MFP shock when all of the conditions for aggregate equivalence are met except that adjustment costs depend on a mixture of units as in some recent formulations. In particular, v_2^E is set equal to 0 but v_1^E is left equal to 1. Specifying adjustment costs in this alternative way temporarily lowers the cost of adjustment relative to the specification that reflects only efficiency units. The difference is largest in the first period. The first comparison confirms that the specification of investment adjustment costs can, by itself, break aggregate equivalence.
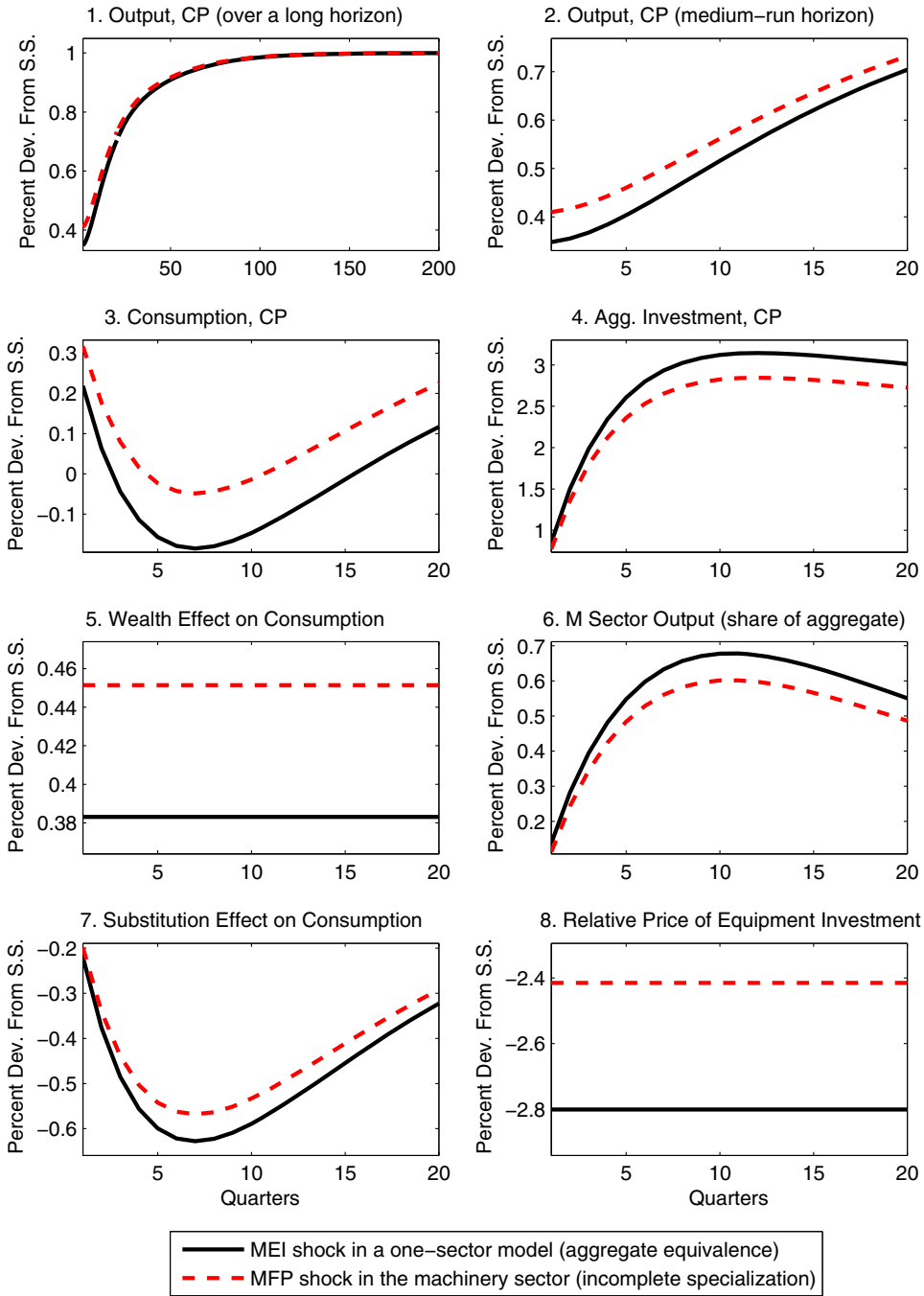
4.2. *Alternative Calibration with All Departures.* The second comparison in Figure 3 involves the solid and dashed lines. Recall that the solid lines show results with the baseline calibration. The dashed lines record results for a machinery-sector MFP shock under the alternative calibration reported in Table 4. The alternative calibration departs from the baseline
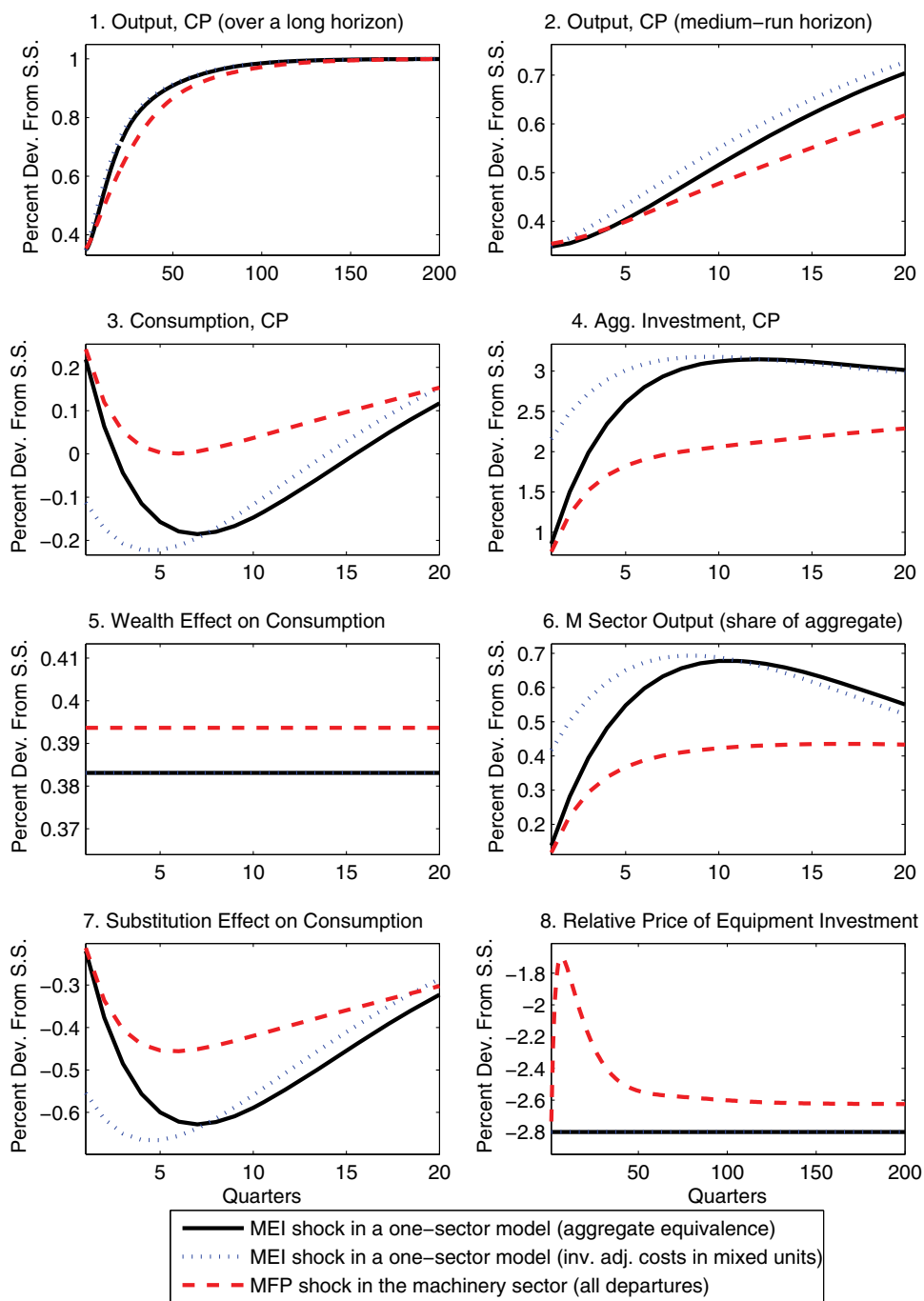
NOTE: With aggregate equivalence, investment adjustment costs are specified in efficiency units. The dotted lines show the effects of a machinery-sector MFP shock when all of the conditions for aggregate equivalence are met except that investment adjustment costs depend on a mixture of units. See discussion in Section 4.1. CP stands for "at constant prices."

FIGURE 3

CUMULATIVE EFFECTS OF ALL DEPARTURES FROM AGGREGATE EQUIVALENCE

NOTE: CP stands for "at constant prices."

FIGURE 4

MEI SHOCK UNDER BASELINE CALIBRATION AND SECTORAL MFP SHOCK WITH INCOMPLETE SPECIALIZATION IN ASSEMBLY

TABLE 4
ALTERNATIVE CALIBRATION REFLECTING ALL DEPARTURES FROM AGGREGATE EQUIVALENCE

Parameter	Determines	Parameter	Determines
	Adaptation Costs		
$\omega^E = 100$	M, N Equipment Capital	$\omega^S = 100$	M, N Structures Capital
	M Goods Production		
$\alpha_M^N = 0.46$	Labor share	$\alpha_M^E = 0.43$	Equipment share
$\alpha_M^S = 0.11$	Structures share		
	N Goods Production		
$\alpha_N^N = 0.72$	Labor share	$\alpha_N^E = 0.15$	Equipment share
$\alpha_N^S = 0.13$	Structures share		
	Consumption Assembly		
$\phi_M^C = 0.04$	M goods intensity	$\phi_N^C = .96$	N goods intensity
$\sigma_C = 0.5$	Substitution elast. for M and N goods		
	Assembly of Equipment Investment		
$\phi_M^E = .85$	M goods intensity	$\phi_N^E = .15$	N goods intensity
$\sigma_E = 0.5$	Substitution elast. for M and N goods		
	Assembly of Structures Investment		
$\phi_M^S = 0.04$	M goods intensity	$\phi_N^S = .96$	N goods intensity
$\sigma_S = 0.5$	Substitution elast. for M and N goods		

NOTES: For ease of comparison with Table 3, this table only reports the parameters that vary from the baseline calibration.

calibration in three essential ways as described below. In order to highlight the importance of these departures, aggregate factor shares are kept the same as in the baseline calibration.

1. Predetermined capital stocks
   By setting $\omega^E = \omega^S = 100$ capital stocks become essentially predetermined in each sector as well as at the aggregate level.
2. Sector-specific production functions
   Following Greenwood et al. (1997), the baseline calibration implies identical production functions across sectors. However, for the three factor inputs in the model, U.S. data imply different input intensities across the machinery and non-machinery sectors (the M and N sectors in the model).
   To differentiate the intensities of factor inputs across sectors, we used the following restrictions: (a) while allowing variation across sectors, we kept the aggregate factor input intensities the same as in Greenwood et al. (1997); (b) factor payments are equalized across sectors, making the factors' shares of sectoral output proportional to the sectoral stocks of capital (since production functions are Cobb–Douglas)[30]; (c) factor input intensities are equal regardless of where the output of a sector is used.
   We combined data for the net capital stock of private nonresidential fixed assets from the U.S. Bureau of Economic Analysis, with data from the Input–Output Bridge Table for Private Equipment and Software. The first data set contains data on the size of equipment and nonequipment capital stocks by sector. The second data set allowed us to ascertain

[30] If capital stocks are predetermined at the sectoral level, rentals are equalized only in the long run.

the commodity composition of private equipment and software. Finally, we used BEA data to establish a sector's value added output. We focused on the year 2004, the latest available at the time of writing, but similar sector-specific production functions would be implied by older vintages of data.

Our calculations show that the machinery-producing sector is less intensive in structures and labor than the aggregate economy, but more intensive in equipment capital. For the machinery sector, the share of structures is 11%, the labor share 46%, and the share of equipment capital the remaining 43% (thus, $\alpha_M^S = 0.11$, $\alpha_M^N = 0.46$, $\alpha_M^E = 0.43$). For the nonmachinery sector the share of structures is 13%, the share of labor 72%, and the share of equipment capital 15%.

3. Incomplete specialization

The baseline calibration assumes complete specialization in the assembly of investment and consumption goods. Equipment investment is assembled using output from the M sector only. In contrast, structures investment and consumption goods are assembled using output from the N sector only. This complete specialization does not reflect the composition of final goods revealed in the Input-Output Bridge Tables that link final uses in the NIPA to sectors (industries) in the U.S. Input-Output Tables. For example, according to the data for 2004, wholesale and retail services (part of our nonmachinery sector) are important inputs not only for consumption but also for equipment investment, accounting for 15% of the total output of private equipment and software.[31] Furthermore, electric and electronic products are used in the assembly of consumption, accounting for 4% of the total.[32]

The model captures the commingling implied by the bridge tables through assembly functions that specify how inputs from the M and N sectors are combined to obtain consumption, structures investment, and equipment investment. In the alternative calibration used to generate the dashed lines in Figure 3, the share parameters for the assembly functions are set as follows: The shares for equipment investment are $\phi_M^E = 0.85$, $\phi_N^E = 0.15$ and the shares for consumption and structures investment are $\phi_M^C = \phi_M^S = 0.04$, $\phi_N^C = \phi_N^S = 0.96$. We assume that in each of the final-good assembly functions the elasticity of substitution between inputs from the M and N sectors is 0.5 (i.e., $\sigma_C = \sigma_E = \sigma_S = 0.5$). This relatively low substitution elasticity seems appropriate given that the assembly functions capture the commingling of inputs as different as electronic equipment on one side and wholesale, retail, and transportation services on the other.

4.3. *The Effects of MFP Shocks under the Alternative Calibration.* Some key differences between the implications of MFP and MEI shocks can be highlighted by decomposing the responses of consumption into substitution and wealth effects. The bottom left panels of Figure 3 show the Hicksian decomposition laid out by King (1991) for general equilibrium models. Given our isoelastic utility function, the change in utility ΔU is computed in the following way:

$$(19) \qquad \Delta U = E_t \sum_{t=0}^{\infty} \beta^t \bar{C}^{1-\gamma} \hat{C}_t,$$

[31] There are bridge tables for consumption as well as equipment and software investment but not for structures investment. We assume that the sectoral composition of structures investment is the same as that of consumption.

[32] The machinery sector of our model has two components. The first component is the NIPA definition of "Equipment and Software" Investment, after excluding the Transportation, Wholesale, and Retail Margins from the IO Tables. Most of the industries whose output is used in "Equipment and Software" produce exclusively for "Equipment and Software." The second component of our machinery sector comprises those inputs for consumption assembly from all the industries that produce inputs used in both the NIPA definition of "Equipment and Software" Investment and of "Consumption." These IO Table industries are (334) Computer and Electronic Products; (335) Electrical Equipment, Appliances, and Components; (513) Broadcasting and Telecommunications; (514) Information and Data Processing Services; and (5412OP) Miscellaneous Professional, Scientific and Technical Services.

where a bar indicates a steady state and a caret symbol denotes a log deviation from that steady state. The wealth effect on consumption is given by the log change in steady-state consumption that would yield the same change in utility as that generated by the shock, whereas the real interest rate is kept constant at its steady-state value. Accordingly, the Euler equation for consumption implies that the wealth effect on consumption $\hat{\bar{C}}$ is constant over time and equal to

$$(20) \qquad \hat{\bar{C}} = (1 - \beta) \frac{\Delta U}{\bar{C}^{1-\gamma}}.$$

The substitution effect is the path of consumption that would induce no change in utility in reaction to the interest rate changes induced by the shock. Accordingly, the substitution effect on consumption, $\hat{\tilde{C}}_t$, solves the system

$$(21) \qquad 0 = E_t \sum_{t=0}^{\infty} \beta^t C^{1-\gamma} \hat{\tilde{C}}_t,$$

$$(22) \qquad E_t \hat{\tilde{C}}_{t+1} = \hat{\tilde{C}}_t + \frac{1}{\gamma} \hat{R}_t,$$

where $\hat{R}_t$ is expressed as the difference of the interest rate from its steady-state value. Combining Equation (22) and Equation (21) yields the result that $\hat{\tilde{C}}_0 = -\frac{\beta}{\gamma} \sum_{t=0}^{\infty} \beta^t \hat{R}_t$, which allows one to solve for the full path of the substitution effect by combining knowledge of $\hat{\tilde{C}}_0$ with Equation (22) above.

A common feature among changes implied by the alternative calibration is a reduction in the magnitude of the substitution effect on consumption associated with the MFP shock. With capital predetermined at the sectoral level, more of the factor inputs remain temporarily locked up in the N sector, reducing the substitution effect associated with the MFP shock.[33] This reduction dampens the response of consumption, as its composition is intensive in the output of the N sector. Similarly, structures and equipment capital take longer to shift back and forth across sectors, making the response of aggregate investment more subdued.

Under the alternative calibration, with sector-specific production functions, the making of M-sector goods used in equipment investment is more intensive in equipment capital relative to the aggregate. This feature helps explain why the substitution effect on consumption is smaller with the MFP shock than with the MEI shock. Accordingly, M-sector output and investment increase by less at first.

Finally, the incomplete specialization in the assembly of equipment investment not only reduces the magnitude of the substitution effect but also boosts the wealth effect. Relaxing the assumption of complete sectoral specialization implies that the MFP shock in the M sector acquires a direct effect on consumption through the assembly function.

Altogether, the weaker substitution effect and stronger wealth effect lead to a uniform rise in consumption in reaction to the MFP shock (whereas consumption temporarily falls for the MEI shock) and a corresponding reduction in the rise of investment relative to the effects of the MEI shock. The combined effect of all the departures from the baseline calibration is to generate qualitative differences between the consumption responses to MFP and MEI and shocks, as can be seen by comparing the dashed and dotted lines in Figure 3. Although the response to the MFP shock in the machinery sector in the two-sector model is such that consumption never falls, in response to an MEI shock in the one-sector model consumption falls for many quarters beginning soon after the shock. Similarly, aggregate investment shows protracted differences,

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[33] Only with much higher input substitution elasticities would there be an incentive to shift so much labor to the M sector as to lower the output of the N sector.

with the response to the MEI shock in the one-sector model being persistently more pronounced than the response to the MFP shock in the two-sector model.

Finally, this alternative calibration also causes a decoupling of the responses of the relative price of investment and the size of the MFP shock. As can be seen in the bottom right panel of Figure 3, the relative price of investment ceases to be the mirror image of the unit-root process for the MFP shock in the two-sector model. Except early in the horizon, the drop in the relative price of investment is not as pronounced as the long-run drop due to elevated demand for equipment investment. Under this scenario, using the relative price of investment to back out MEI shocks would be inappropriate.

4.4. *Isolating the Role of Incomplete Specialization.*   Although all of the departures from the baseline aggregate calibration are important in reversing the conditional correlation between consumption and investment implied by MFP shocks in the machinery sector, a key role is played by incomplete sectoral specialization in the assembly of final goods.[34] Figure 4 compares again the effects of an MFP shock in a two-sector model with those of an MEI shock in a one-sector model. The solid lines denoting the effects of the MEI shock replicate what is shown in Figure 1. The calibration used in constructing the effects of the MFP shock in the *M* sector departs from the aggregate equivalence calibration summarized in Table 3 only insofar as it allows for incomplete sectoral specialization in the assembly of final goods, as described in Section 4.2. With the baseline calibration for investment adjustment costs, this change alone is sufficient to reverse the short-term correlation between investment and consumption. For example, over the first ten quarters, consumption falls on net under the baseline calibration but rises on net under the alternative.

5.   CONCLUSION

Under stringent conditions, MFP shocks in a two-sector model and MEI shocks in a one-sector model have the same effects on aggregate variables, a result that we dub "aggregate equivalence." Revisiting these conditions, we extend them to take account of adjustment costs for investment, different intensities for factor inputs across production sectors, the sectoral composition of outputs used in the assembly of final goods, and costs of adapting capital for use in different sectors.

We present impulse responses for two calibrations of our two-sector model. One is a "base-line calibration" that satisfies our extended conditions for aggregate equivalence. The other is an "alternative calibration" with three important departures. The first is that capital stocks are predetermined at the sectoral level. The second and third departures reflect the input–output data: Sectoral production functions have different factor intensities, and assembly functions have incomplete specialization. We compare the effects of a machinery-sector MFP shock under the alternative calibration with those of an MEI shock under the baseline calibration. There is a striking qualitative difference between the results: In the first several periods, investment and consumption both rise with the MFP shock, whereas investment rises and consumption falls with the MEI shock. Although investment adjustment costs are not sufficient by themselves for obtaining comovement, it is shown in Section A.2 of the Appendix that they are necessary under our calibration. Given that investment adjustment costs are present, as in our baseline calibration, incomplete specialization in assembly is sufficient by itself to generate such qualitatively different effects for sectoral MFP shocks and MEI shocks.

With some stringent restrictions a two-sector model with MFP shocks is equivalent to a one-sector model with MEI shocks. Abstracting from sectoral implications, imposing these restrictions has implications that are hard to square with the aggregate data on consumption

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[34] In Guerrieri et al. (2010) we isolate the role of capital being predetermined at the sectoral level and of sector-specific production functions in generating differences between the effects of MFP shocks in the machinery sector and those of MEI shocks in a one-sector model.

and investment. We conclude that a model in the spirit of our extended model with two (or more) sectors and without these restrictions is likely to be more useful in analyzing the effects of sector-specific productivity shocks even when the focus is on aggregate outcomes.

APPENDIX

Section A.1 provides support for the assertions made in Section 3.1. Section A.2 contains additional simulation results.

A.1. *Equivalence.*

A.1.1. *Sufficiency of set A conditions for 2SE.*   It can be shown that the equations of the model can be written in a form such that when the set A conditions are imposed Z and A always enter together in the form $Z_s(A_s)^{\phi_M^E}$. For example, Equation (16), repeated here for convenience,

$$D_{is}^E = \left(1 - \delta^E\right) K_{is-1}^E + Z_{s-1} \left(A_{s-1}\right)^{\phi_M^E} j_{is-1}^E \bar{J}_{s-1}^E - \frac{v_{0i}^E}{2} \left(Z_{s-1}\right)^{v_1^E} \left(A_{s-1}\right)^{\phi_M^E v_1^E} j_{is-1}^E \bar{J}_{s-1}^E$$

$$\times \left[\left(\frac{Z_{s-1}}{Z_{s-2}}\right)^{v_2^E} \left(\frac{A_{s-1}}{A_{s-2}}\right)^{\phi_M^E v_2^E} \frac{j_{is-1}^E \bar{J}_{s-1}^E}{j_{is-2}^E \bar{J}_{s-2}^E} - 1 \right]^2, \quad i \in \{M, N\},$$

satisfies the set A conditions either when $v_1^E = v_2^E = 1$ or when $v_1^E = v_2^E = 0$.

A.1.2. *Necessity of set A conditions for 2SE.*   2SE in the full model implies 2SE for the model approximated to first order. We show that set A conditions are necessary for 2SE in the first-order approximation of the model. Therefore, they must also be necessary for 2SE in the full nonlinear model. To support the assertion that the set A conditions are necessary for 2SE to first order, linearize the unrestricted equations of the model around a steady state. The combinations of shocks that yield equivalent outcomes are obtained by setting the changes for all the endogenous variables equal to zero for all periods. Consider an arbitrary sequence of changes in the MFP shock A_s, $s \in \{0, \infty\}$. Confirm that the zero-change equilibrium conditions can be satisfied only if terms in changes in A_s and terms in changes in Z_s always appear together in the same linear combination. The necessity of condition A-1 is established by noting that if A-1 is not met, A_s enters the assembly function for at least consumption or structures investment but Z_s does not enter either. That A-2 and A-3 are necessary is established by showing that a single linear combination of changes in Z_s and changes in A_s would not satisfy some set of equations. For A-2, the set comprises the equipment assembly function and the first-order conditions for cost minimization in equipment assembly (not included in the article). For A-3, the set comprises the equipment assembly function and the accumulation equations for equipment capital stocks.

A.1.3. *Sufficiency of adding set B conditions for AE.*   As stated in the text, Greenwood et al. (2000) and Oulton (2007) have shown that conditions B-1 through B-3 are sufficient for aggregate equivalence in models without investment adjustment costs. If condition B-4 is imposed, sectoral capital stocks, investment flows, and capital accumulation equations can be combined to yield the definitional equations for aggregate variables:

(A.1) $$K_s^S = K_{Ms}^S + K_{Ns}^S, \quad K_s^E = K_{Ms}^E + K_{Ns}^E,$$

(A.2) $$J_s^S = J_{Ms}^S + J_{Ns}^S, \quad J_s^E = J_{Ms}^E + J_{Ns}^E.$$

Using Equations (14) and (15), one can derive the laws of motion for the aggregate capital stocks:

$$(A.3) \qquad K_s^S = \left(1 - \delta^S\right) K_{s-1}^S + J_{s-1}^S - \frac{v_0^S}{2} J_{s-1}^S \left(\frac{J_{s-1}^S}{J_{s-2}^S} - 1\right)^2,$$

$$(A.4) \qquad K_s^E = \left(1 - \delta^E\right) K_{s-1}^E + Z_{s-1} \left(A_{s-1}\right)^{\phi_M^E} J_{s-1}^E$$

$$- \frac{v_0^E}{2} \left[Z_{s-1} \left(A_{s-1}\right)^{\phi_M^E}\right]^{v^E} J_{s-1}^E \left[\left(\frac{Z_{s-1} \left(A_{s-1}\right)^{\phi_M^E}}{Z_{s-2} \left(A_{s-2}\right)^{\phi_M^E}}\right)^{v^E} \frac{J_{s-1}^E}{J_{s-2}^E} - 1\right]^2,$$

where v^E is equal to either zero or one.

A.2. *Additional Simulation Results.*   The discussion in the main body of the article did not consider in isolation three departures from our baseline calibration: (1) the effects of relaxing perfect capital mobility across sectors; (2) the effects of varying the factor intensities across sectors; and (3) the effects of adjustment costs. The effects of these three departures are illustrated, in turn, below.

In Figure A1, the solid lines reproduce the responses to the MEI shock from Figure 1. Instead, the dashed lines show the economy's response to an MFP shock in the M sector when relaxing only the assumption of perfect capital mobility across sectors in every period. Perfect capital mobility, as argued before, is necessary to represent our two-sector model as an aggregate one-sector model. To produce the responses shown by the dashed lines, we set the parameters governing the capital adjustment costs ω^E and ω^S both equal to 100. This parametrization implies that sectoral capital allocations only move with a delay of one period. Thus capital stocks are not only predetermined at the aggregate level, but also at the sectoral level.

The size of the MFP shock hitting the M sector was again chosen to bring about a permanent 1% increase in aggregate output. Although the wealth effect on consumption is identical for the two shocks in Figure A1, the negative substitution effect is reduced in magnitude when the sectoral capital stocks are predetermined.

Figure A2 shows the responses to an MEI shock in the aggregate model (replicating, for ease of comparison, what is also shown in Figures 1 and A1), as well as the responses to an MFP shock in the machinery sector of a two-sector model that allows for sector-specific production functions (the only difference relative to the baseline calibration). Again, the magnitude of the MFP shock is chosen to match the 1% long-run increase in aggregate output for the MEI shock.

The figure shows persistent differences in the responses of consumption and investment. As under the alternative calibration, the making of M-sector goods used in equipment investment is more intensive in equipment capital relative to the aggregate, the substitution effect on consumption coming from the MFP shock is not as strong initially relative to the MEI shock. Accordingly, M-sector output increases by less, at first. However, eventually more resources need to be devoted to the M sector to maintain the larger stock of equipment capital implied by the alternative calibration, and the MFP shock in the investment sector leads to a larger long-run increase in equipment investment and a smaller long-run increase in consumption. Consequently, the wealth effect on consumption is smaller for the MFP shock than for the MEI shock.

High adjustment costs for investment, by slowing adjustment, have the potential to dampen the negative correlation between consumption and investment following MEI and sector-specific MFP shocks. To investigate the importance of investment adjustment costs in preventing consumption from falling after a sector-specific MFP shock, Figure A3 presents
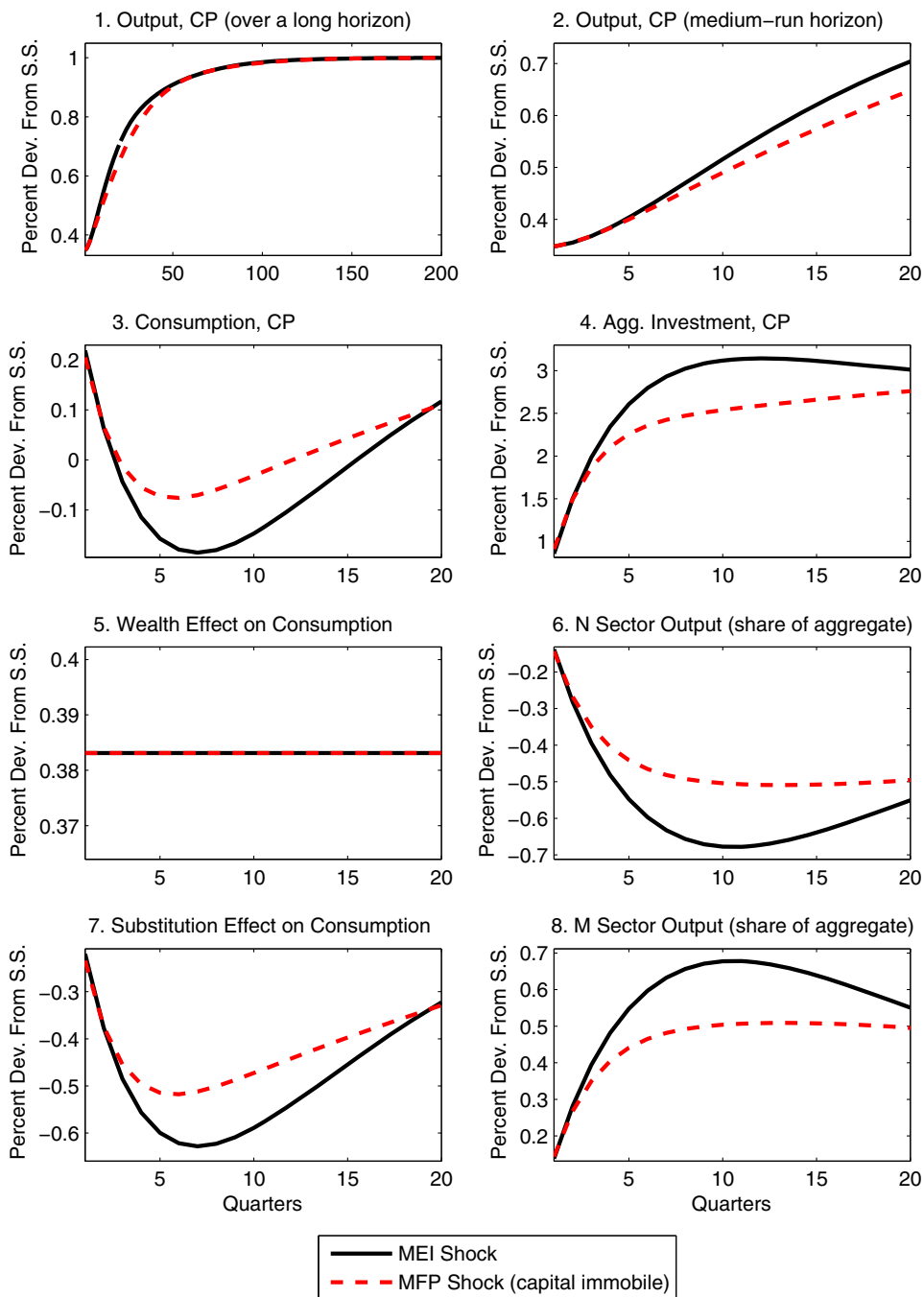
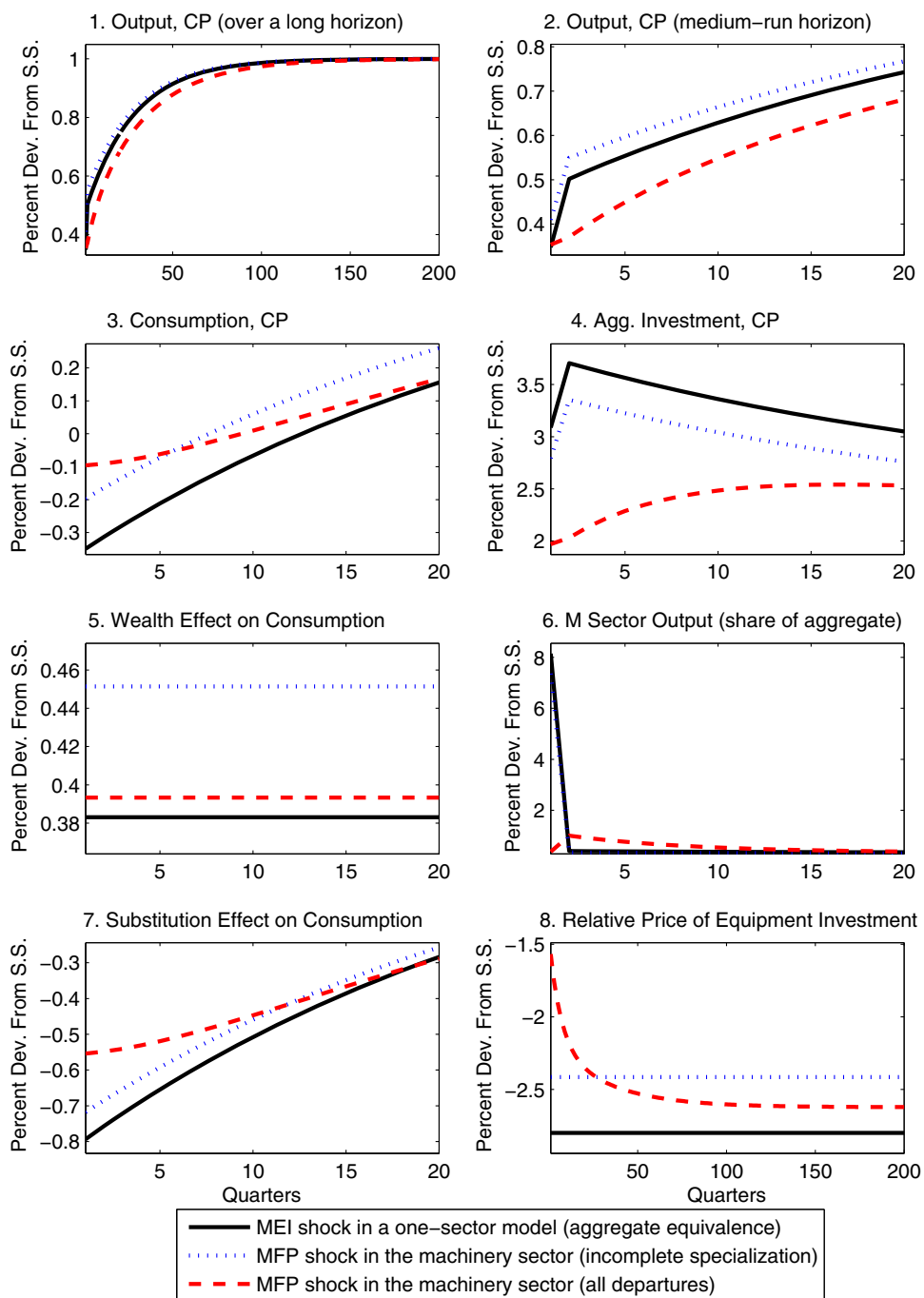NOTE: CP stands for "at constant prices."

FIGURE A1

MEI UNDER BASELINE CALIBRATION AND MFP SHOCK WITH CAPITAL STOCKS PREDETERMINED IN EACH SECTOR
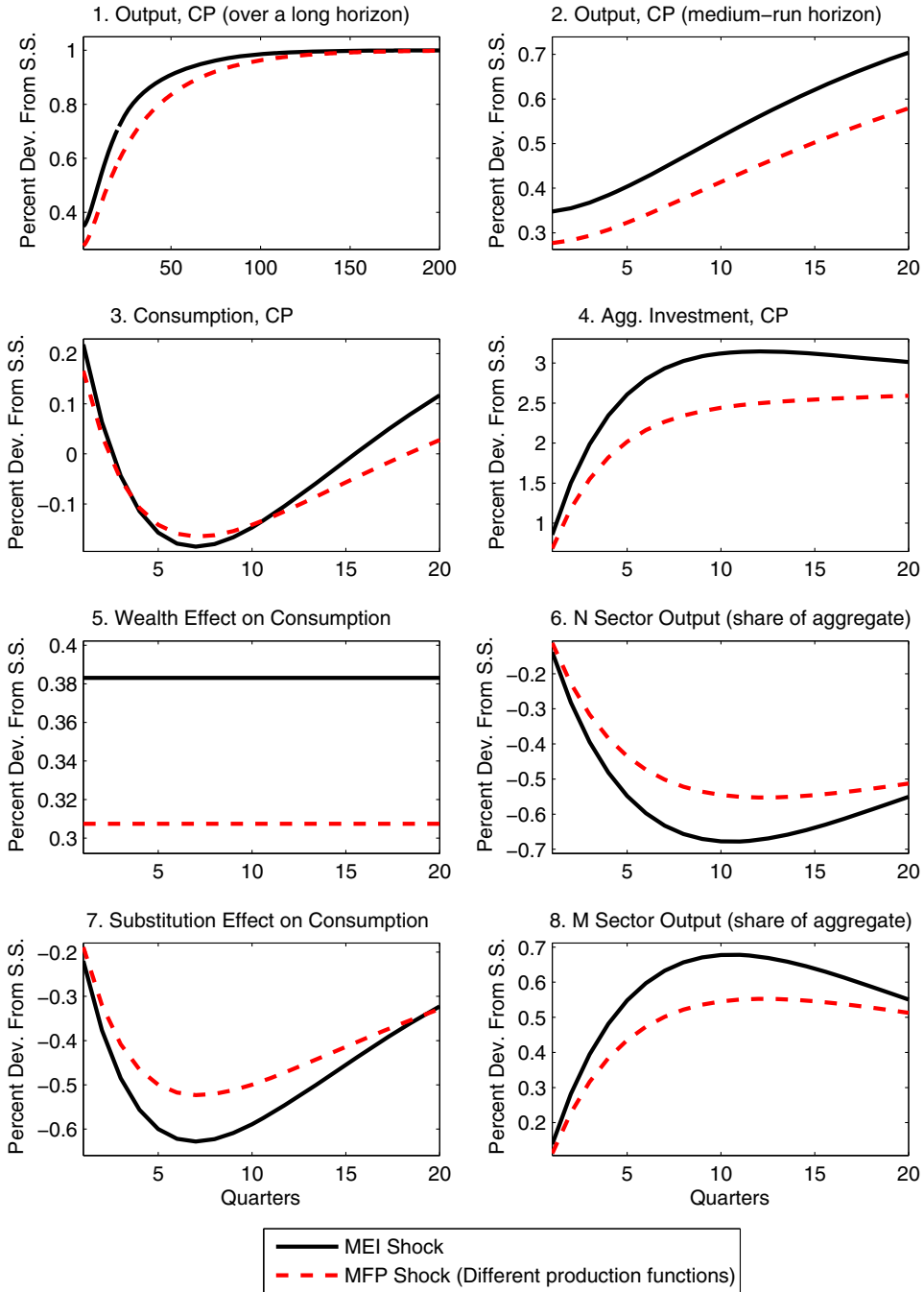
NOTE: CP stands for "at constant prices."

FIGURE A2

MEI UNDER BASELINE CALIBRATION AND MFP SHOCKS WITH SECTOR-SPECIFIC PRODUCTION FUNCTIONS

NOTE: CP stands for "at constant prices."

FIGURE A3

SENSITIVITY ANALYSIS: NO INVESTMENT ADJUSTMENT COSTS

simulations that abstract from such costs.

The solid line shows the effects of a sectoral MFP (or an MEI) shock with aggregate equivalence. We depart from the calibration described in Table 3 only insofar as we eliminate the investment adjustment costs by setting $v_0^E = v_0^S = 0$. As investment can now jump on impact, the negative correlation between consumption and investment becomes stronger.

The dashed lines show the effects of an MFP shock with all departures from the assumptions for aggregate equivalence except investment adjustment costs. Without adjustment costs, consumption falls initially even with all the other departures. That is, given our calibration, adjustment costs are necessary for comovement.

The dotted line in Figure A3 shows the responses to a sectoral MFP shock when the only assumption that departs from those for aggregate equivalence is incomplete specialization. It shows that consumption turns negative on impact. This simulation shows that incomplete specialization plays an important quantitative role in reducing the negative correlation between consumption and investment following shocks to the equipment-producing sector. However, incomplete specialization alone cannot reverse the initial negative correlation between consumption and investment without adjustment costs. Furthermore, the simulation confirms that no single departure from the conditions for aggregate equivalence—by itself—can account for the positive comovement between investment and consumption conditional on sector-specific MFP shocks.

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