Interpreting Shocks to the Relative Price of Investment with a Two-Sector Model*

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Abstract

Consumption and investment comove over the business cycle in response to shocks that permanently move the price of investment. The interpretation of these shocks has relied on standard one-sector models or on models with two or more sectors that can be aggregated. We show that the same interpretation can also be motivated with models that cannot be aggregated into a standard one-sector model. Furthermore, such a two-sector model with distinct factor input shares across production sectors and commingling of sectoral outputs in the assembly of final consumption and investment goods, in line with the U.S. Input-Output Tables, has implications not only for sectoral variables but also for aggregate variables. Namely, it yields a closer match to the empirical evidence of positive comovement for consumption and investment subject shocks that permanently move the price of investment.

Keywords: DSGE model, VAR, long-run restrictions.

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1. Introduction

Fisher (2006) used a VAR identified with long-run restrictions to show that shocks to the relative price of investment can explain more than 70% of the fluctuations in hours worked over the business cycle. To interpret the permanent shock to the relative price of investment, Fisher (2006) relied on a one-sector model with investment-specific technology (IST) shocks that increase the efficiency of investment in a capital accumulation equation. We show that the identification scheme of Fisher also applies to full-fledged two-sector models with sector-specific multi-factor productivity (MFP) shocks.

In particular, this paper makes three contributions: 1) We show analytically and confirm numerically that the long-run identification scheme proposed by Fisher is consistent with a general two-sector model; 2) Extending the VAR estimated by Fisher (2006) to include household consumption and investment, we document thoroughly that the unconditional positive correlation between consumption and investment emphasized by other papers continues to be positive also when conditioning on shocks that move the price of investment permanently; 3) As an application of our theoretical results, a Monte Carlo experiment indicates that that sectoral MFP shocks are more likely to be consistent with the positive conditional correlation uncovered from the VAR than IST shocks. We elaborate on each of these contributions below.

Regarding our first contribution, an expanding literature starting with Greenwood, Hercowitz, and Krusell (1997) has underscored the importance of sectoral productivity developments to explain both trends and cycles. Much of the empirical evidence is tied to strong theoretical assumptions implicit in the use of DSGE models. The much cited contribution of Fisher (2006) quantified the importance of sectoral productivity developments abstracting from many strong theoretical assumptions. However, the interpretation of his identification scheme still relied on an aggregate one-sector model. Guerrieri, Henderson, and Kim (2014) characterized the conditions under which a multi-sector model can be reduced to an aggregate one-sector model. Those conditions include equality of input factor shares across sectors, a condition at odds with evidence from the U.S. Input-Output Tables. Accordingly, our previous results might seem to invalidate Fisher's identification scheme, since his motivation was based on empirically irrelevant assumptions. To wit, Basu, Fernald, Fisher, and Kimball (2013) argued that the identification scheme of Fisher (2006) does not apply when sectoral production functions display different factor intensities. The theorem in this paper shows that, in fact, the identification scheme in Fisher (2006) is more general than Fisher's original motivation indicates and applies to multi-sector models that have different factor input shares across sectors in line with the Input-Output Tables.

Several other papers showed that it is difficult to reconcile the importance of sectoral technology shocks with the observation that aggregate consumption and investment co-move (unconditionally).[1] For our second contribution, we document that we can also expect comovement between consumption and investment not just unconditionally, but also conditionally on shocks that move the relative price of investment permanently. Fisher (2006) provided a link between VAR evidence and DSGE results, but did not include consumption or investment measures in the VAR specification (only the relative price of investment).[2] To our knowledge, we are the first to document thoroughly this type of conditional comovement using a VAR.

Others have shown that various economic mechanisms can augment a stylized DSGE model to yield positive comovement between consumption and investment (unconditionally) when sectoral technology shocks are a prominent source of fluctuations. As a third contribution, we show that a different (and possibly more fundamental) mechanism that relies on empirically relevant sectoral differences can generate comovement consistent with our VAR results. Guerrieri, Henderson, and Kim (2014) also examined this possibility, but did not substantiate its empirical relevance with a link to evidence from a VAR and a Monte Carlo experiment, as we do here.

We proceed by considering two alternative DSGE models based on Guerrieri, Henderson, and Kim (2014): A) a two-sector model with imperfect sectoral specialization in the production of final goods, in which sectoral MFP shocks and IST shocks would not coincide—we dub this model the MFP model; and B) a model with full specialization in the assembly of final goods, in which IST shocks can be interpreted as MFP shocks in the investment-producing sector, as in Greenwood, Hercowitz, and Krusell (1997)—we dub this model the IST model. Both models have two production sectors, a machinery-producing sector and its complement that is dubbed a non-machinery-producing sector. They allow for the assembly of consumption and investment goods each using sectoral outputs in different proportions. In the first model, the proportions of machinery and non-machinery goods

[1] Christiano and Fitzgerald (1998) provide a good overview of the literature on comovement. The mechanisms suggested by Greenwood, Hercowitz, and Krusell (2000) revolves around variable capacity utilization for capital. Christiano, Ilut, Motto, and Rostagno (2008) point to strong consumption habits and investment adjustment costs as a mechanism for generating comovement. Jaimovich and Rebelo (2009) focus on departures from utility functions that are additively separable in consumption and leisure to generate comovement.

[2] A working paper version of Fisher (2006) did include consumption and investment (as a share of GDP) in the VAR, but did not emphasize or quantify their correlation at business cycle frequencies in response to shocks that permanently move the price of investment.

used to produce consumption and investment reflect the U.S. Input-Output Tables and other sectoral statistics. This imperfect specialization in the production of final goods prevents the reinterpretation of IST shocks as MFP shocks at the sectoral level (as shown by Guerrieri, Henderson, and Kim (2014)). By contrast the second model with full specialization makes this reinterpretation viable.

To interpret the permanent shock to the relative price of investment identified from the VAR, Fisher (2006) focused on a model with IST shocks that increase the efficiency of investment in a capital accumulation equation.[3] This approach is based on the results of Greenwood, Hercowitz, and Krusell (2000), who showed that, under certain conditions, a two-sector model with an MFP shock in each sector can be recast as an aggregate model with IST shocks as well as neutral MFP shocks.[4] These conditions include equal factor shares across production sectors, assembly of each final good using the output of a single production sector, and perfect mobility of capital across production sectors.[5] Nonetheless, much of the literature has proceeded with an aggregate approach.

Because the sectoral production functions display different factor intensities, aggregation is not possible in our two-sector model. In Section 4 we prove analytically that relative prices are still informative about sectoral productivity developments even if factor input shares and depreciation rates are different across sectors. Furthermore, our proof also allows for imperfect specialization in the assembly of final goods.

When our two models are estimated to match the same aggregate features, the MFP model and the IST model have different implications for aggregate variables. One important difference is that, conditional on shocks that move the price of investment permanently, the correlation between consumption and investment is higher for the MFP model, allowing that model to better match the corresponding estimates from VAR. The commingling of sectoral outputs in the assembly of both consumption and investment goods implies that an increase in productivity in one production sector lowers the cost of assembly of both final goods, creating an incentive to increase the assembly of both goods.

[3] Productivity developments at the sectoral level are not the only possible source of long-run fluctuations in the relative price of investment. Permanent capital tax rate shocks, for instance, could also be a source of these movements. Nonetheless, the depreciation allowance in the U.S. tax code greatly diminishes the applicable tax base, pointing to a relatively small influence of these shocks on relative prices.

[4] Guerrieri, Henderson, and Kim (2014) referred to shocks that influences a capital accumulation equation in a general two-sector model as marginal efficiency of investment (MEI) shocks. It is possible to identify both MEI shocks and MFP shocks if the relevant data on the relative price of investment are available. Justiniano et al. (2011) identified the two shocks separately by taking a stance on how well the available investment price series accommodate hedonic adjustments.

[5] According to the U.S. Input-Output Tables, different production sectors display different intensities of factor inputs and assembly of each final good uses outputs from more than one production sector. Moreover, as shown, for example, by Ramey and Shapiro (1998) it is quite costly to move capital across sectors.

The median estimate of this conditional correlation is 0.95, and the 90 percent confidence interval ranges from 0.55 to 0.99 for our baseline sample (from 1982:Q3 to 2008:Q3). We show that these estimates are robust to alternative samples and alternative definitions of investment (we also consider a measure of investment that include consumption durables). Heretofore, empirical results have hinted at a positive correlation. For instance, Fisher (2006) found that these shocks account for such a large fraction of business cycle movements in aggregate variables that a negative conditional correlation between consumption and equipment investment would be unlikely. Nonetheless, we are not aware of other work that has quantified this important statistic using a VAR identified with long-run restrictions.[6]

The imprecision of estimates from long-run identification strategies applied to small samples can make it difficult to discriminate between alternative hypotheses.[7] To investigate the small sample properties of the VAR estimates, we rely on a Monte Carlo experiment. We re-estimate the same VAR used on observed U.S. data on random samples of data generated from the two alternative models. The cumulative density function for the correlation between consumption and investment for the MFP model is uniformly closer to that for the empirical VAR, confirming that model as a more plausible candidate data-generating process than the IST model for this correlation pattern.

The tables are turned when it comes to neutral MFP shocks (shocks that move labor productivity permanently but not relative prices). The VAR evidence for the correlation between consumption and investment at business cycle frequencies is not strong for our baseline sample. The median correlation implied by the VAR conditional on neutral shocks is about 0.5 and a 90 percent confidence interval ranges all the way from -0.80 to 0.95 (though alternative samples support tighter estimates of strongly positive correlation). VARs estimated on data drawn from our two models also show substantial uncertainty for this moment conditional on the baseline sample size.[8] Nonetheless, the Monte Carlo results point to the aggregate IST model as yielding a closer match to the data in this respect.

One way to weigh these two opposite results for sectoral and neutral productivity shocks is by

[6] Using sign restrictions motivated from a DSGE model, Peersman and Straub (2006) found that consumption initially increases in response to an "investment" shock that they model as a change in the adjustment cost of investment, but they do not focus on correlations.

[7] See, for instance, Faust and Leeper (1997) and Erceg, Guerrieri, and Gust (2005) for an examination of the econometric issues related to long-run restriction schemes. Beyond long-run schemes, Cooley and Dwyer (1998) provide a comprehensive list of additional problems that can arise when VAR evidence is used to validate DSGE models.

[8] The signal of the smaller neutral productivity shock, relative to the sectoral productivity shock, is eclipsed more easily by other non-productivity shocks in a short sample.

the relative importance of each shock in driving business cycle fluctuations. As the VAR evidence points to the sectoral shocks as most prominent, on balance, we still view the sectoral IST model as providing a better match to the data. Of course, in addition, the sectoral MFP model also has the virtue of reflecting important features of the production structure captured by the U.S. Input-Output Tables.

2. New Empirical Evidence on the Correlation Between Consumption and Investment

A key feature for discriminating between a one-sector model with IST shocks and a two-sector model with MFP shocks is the comovement of consumption and investment conditional on technology shocks. Fisher's important work on identifying IST shocks did not include measures of consumption or investment in the VAR, making it impossible to investigate this comovement. We update Fisher's results and extend them to gauge this comovement by including measures of consumption and investment in the VAR.

The VAR that we estimate includes five variables:

1. the growth rate of the relative price of investment, constructed as the log-differenced implicit price deflator for equipment and software from NIPA Table 1.1.9 minus log-difference non-farm business output prices (net of equipment and software using the Laspeyres formula);[9]

2. labor productivity growth, measured as log-differenced labor productivity in the nonfarm business sector from the Bureau of Labor Statistics;

3. hours per capita, constructed as the log of hours worked in the nonfarm business sector minus the log of civilian non-institutional population 16 years and over from the Current Population Survey;

4. the growth rate of real equipment and software per capita, defined as the log-differenced equipment and software (nominal equipment and software divided by its implicit deflator) minus the log-differenced civilian non-institutional population 16 years and over from the Current Population Survey;

5. the growth rate of real consumption per capita, constructed as the log-differenced real personal consumption expenditures from NIPA Table 1.1.6, minus the log-differenced civilian

[9] Throughout the body of this paper, we take "investment" to mean investment in equipment and software.

non-institutional population 16 years and over from the Current Population Survey.

We estimate a VAR of order four. The start date for the estimation sample is 1982:Q3, avoiding the adjustment from the Volcker disinflation. We end the sample in 2008:Q3 to avoid a possible break associated with the zero lower bound on nominal interest rates. In robustness analysis, we also consider a longer sample, spanning all available data.

We follow the long-run identification scheme of Fisher (2006). Building on the idea of Greenwood, Hercowitz, and Krusell (2000) that relative prices are informative about sectoral technological developments, Fisher also focused on relative prices. However, to resolve the problem that, in the short run, in the presence of real rigidities relative prices can be influenced by non-technology shocks, he considered long-run movements in relative prices. Following Fisher's scheme, the identification scheme we use posits that only a shock to the relative price of investment can move that price permanently. Moreover, the scheme also posits that only shocks to the relative price of investment and to labor productivity can move the level of labor productivity permanently. All other shocks are left unidentified.

The thick dashed lines in Figure 1 show the effects of a one-standard-deviation shock estimated by our VAR to reduce the price of investment permanently. The point estimate for the decline in the relative price is close to 3 percent. The shaded areas denote 90% confidence intervals constructed following Runkle (1987), and based on 1000 bootstrap replications of the data. While the confidence intervals are strikingly large, they exclude a positive response for the relative price of investment, and negative responses for output, consumption (in all but the first period, in which the lower bound for the confidence interval is barely negative), and investment. From the point estimates for the impulse responses, it can be correctly inferred that there is conditional comovement between consumption and investment.

Table 1 offers a decomposition of the variance of the variables included in the VAR on average over the estimation sample. Shocks to the price of investment account for 60% of the variation in the growth rate of the relative price of investment and they also account for more than 70% of the variation in hours worked, in line with the results presented by Fisher (2006) and confirmed with estimates from a DSGE model by Justiniano, Primiceri, and Tambalotti (2010). In addition, the same shocks are important for the variation in the growth of consumption and investment, accounting for 40% and 45% of this variation, respectively.

The top left panel of Figure 2 shows the cumulative density function (CDF) for the correlation

Table 1: Historical Variance Decomposition Implied by the VAR

Shock	Growth of Price of Investment	Growth of Labor Productivity	Hours	Growth of Consumption	Growth of Investment
Price of Investment	0.60	0.10	0.71	0.40	0.45
Neutral MFP	0.10	0.56	0.03	0.04	0.19

Variable definitions can be found in Section 2.

between consumption and investment at business cycle frequencies, conditional on a shock that changes the relative price of investment permanently, as estimated from the VAR on the baseline sample from 1982q3 to 2008q3. The cumulative density function captures the sampling uncertainty for the estimate of the VAR coefficients and is traced from a bootstrap exercise. First, we sample with replacement from the VAR residuals to construct 1000 new synthetic samples of the same length as the original historical sample. Second, we re-estimate the VAR on each synthetic sample. Third, by another bootstrap on the residuals from the VAR estimated on the synthetic samples, we obtain a population estimate for the correlation between consumption and investment at business cycle frequencies, conditional on a shock that changes the relative price of investment permanently.[10] The median correlation is 0.95. The CDF indicates that negative values for the correlation between consumption and investment are an unlikely occurrence.

The middle left panel of Figure 2 shows the same CDF based on a longer sample, spanning the period from 1948q2 to 2015q1, which includes all the publicly available data at the point of writing. The results from the smaller sample appear robust. The median estimate of the conditional correlation between consumption and investment at business cycle frequencies is still a high 0.85, and the CDF still indicates that negative values are unlikely. As further sensitivity analysis, the bottom left panel repeats the analysis for a case in which the consumption of durable goods is split from overall consumption and allocated to investment. This alternative specification also replicates the high correlation from our baseline specification.

The right panels of Figure 2 consider analogous CDFs for the correlation between consumption and investment at business cycle frequencies conditional on neutral productivity shocks (shocks that move labor productivity permanently but not relative prices). The median estimate for the baseline sample is about 0.5 and a 90 percent confidence interval ranges all the way from -0.80 to 0.95. But longer samples sharpen the estimates of this correlation substantially, as can be seen from the middle

[10] The population estimate of the correlation between consumption and investment is obtained on a bootstrapped sample of 1050 observations, ten times as many as in the original sample. We used a bandpass filter to isolate the oscillations with frequencies between 6 and 32 quarters, typically used to define the business cycle.

and bottom right panels. Monte Carlo experiments point to the relatively smaller size of the neutral shocks in making the precision of this conditional correlation especially susceptible to smaller sample sizes.

In sum, our extensions produce estimates of the correlation between consumption and investment that point to significant comovement over the business cycle conditional on shocks that permanently vary the price of investment. This comovement is robust to alternative sample choices. In our baseline sample the analogous correlation based on shocks to labor productivity is not estimated as precisely, but longer samples point to estimates analogous to those conditional on shocks to the price of investment. Moreover, we verified that our extensions do not overturn previously emphasized results on the importance of shocks to the relative price of investment in explaining business cycle fluctuations.

3. The MFP and IST Models

To interpret his identification scheme, Fisher (2006) wrote down a one-sector model with neutral MFP shocks and IST shocks that enter the capital accumulation equation. The work of Greenwood, Hercowitz, and Krusell (2000) implies that Fisher's identification scheme is consistent with a two-sector model under some restrictive assumptions, including equal factor shares across sectors and complete specialization in the assembly of consumption and investment. These assumptions are at odds with the U.S. Input-Output Tables. We show that Fisher's identification scheme is consistent with our extended two-sector models in which these assumptions are relaxed. In this section, we present the MFP and IST models with factor shares that differ across sectors.

Notice that the models also encompass additional non-technology shocks, such as labor supply shocks, which do not affect the model's long-run properties but that allow us to simulate data to be used in the Monte Carlo experiments discussed in Section 5,

3.1. The MFP Model

In period s, the representative household consumes C_s, supplies labor L_s, chooses next period's capital for the machinery sector, K_{Ms+1}, and for the non-machinery sector, K_{Ns+1}, as well as the borrowing level, B_s, so as to maximize the intertemporal utility function

$$\max_{Cs,I_s,K_{Ns+1},K_{Ms+1},B_s} E_t \sum_{s=t}^{\infty} \left[\tilde{\beta}_s \left((1-\eta)\log(C_s - \eta \bar{C}_{s-1}) - \frac{\chi_0}{1+\chi} V_s \left(L_s\right)^{1+\chi} \right) \right]. \tag{1}$$

The term $\tilde{\beta}_s$ denotes the household's time-varying discount factor, while η parameterizes external habit persistence in consumption. The parameter χ governs the household's labor supply elasticity, while χ_0 governs hours worked in the steady state. The household is subject to the labor supply shock V_s, which evolves according to an auto-regressive process

$$\log(V_s) = \rho_V \log(V_{s-1}) + \epsilon_{Vs}, \tag{2}$$

where log denotes the natural logarithm, ρ_V is the parameter governing the persistence of the auto-regressive process and ϵ_{Vs} is a stochastic innovation drawn from a Normal distribution with standard deviation σ_v. In turn, the discount factor is defined as $\tilde{\beta}_s = \frac{1}{\beta_{t-1}} \prod_{z=t-1}^{s-1} \beta_z$, with β_t evolving according to another auto-regressive process

$$\beta_t - \beta = \rho_\beta(\beta_{t-1} - \beta) + \epsilon_{\beta t}. \tag{3}$$

For the process above, ρ_β is the persistence parameter, ϵ_{Vs} is a stochastic innovation drawn from a Normal distribution with standard deviation σ_β, and β is the steady-state discount factor. We interpret this shock process as introducing an exogenous risk premium when discounting expected future utility.

The household optimization problem is subject to the budget constraint

$$W_s L_s + R_{Ms} K_{Ms} + R_{Ns} K_{Ns} + \rho_{s-1} B_{s-1} = P_{Cs} C_s + P_{Is} I_s + B_s, \tag{4}$$

where W_s is the wage rate, R_{Ms} and R_{Ns}, are the rental rates for K_{Ms} and K_{Ns}, repectively, and ρ_s is the gross interest rate paid on previous period's borrowing. On the right-hand side of the constraint, P_{Cs} is the price of final consumption goods and P_{Is} is the price of final investment goods, I_s. The optimization problem is also subject to the law of motion for the accumulation of capital

$$K_{Ms+1} + K_{Ns+1} = (1 - \delta_M) K_{Ms} + (1 - \delta_N) K_{Ns} + I_s - \frac{\nu}{2} I_s \left(\frac{I_s}{I_{s-1}} - 1 \right)^2, \tag{5}$$

where δ_M and δ_N are the depreciation rates for K_{Ms} and K_{Ns}, respectively, andν parameterizes the adjustment costs for investment.

In each sector, perfectly competitive firms minimize production costs to meet demand subject to the technology constraint as reflected in the following Lagrangian problems:

$$\min_{K_{Ms},L_{Ms},P_{Ms}} R_{Ms}K_{Ms} + W_s L_{Ms} + P_{Ms}(Y_{Ms} - K_{Ms}^{\alpha_M} \left(A_{Ms}L_{Ms}\right)^{1-\alpha_M}), \tag{6}$$

$$\min_{K_{Ns},L_{Ns},P_{Ns}} R_{Ns}K_{Ns} + W_s L_{Ns} + P_{Ns}(Y_{Ns} - K_{Ns}^{\alpha_N} \left(A_{Ns}L_{Ns}\right)^{1-\alpha_N}), \tag{7}$$

where α_M and α_N denote the capital intensities in the production of M and N goods, respectively. The sectoral productivity levels A_{Ms} and A_{Ns}evolve according to the following stochastic processes:

$$A_{Ms} = A_{Ms-1} + \epsilon_{Ms} + \epsilon_{As}, \tag{8}$$

$$A_{Ms} = A_{Ms-1} + \epsilon_{As}, \tag{9}$$

where ϵ_{Ms} is a stochastic innovation, drawn from a Normal distribution with standard deviation σ_M, that is specific to productivty in sector M, and where ϵ_{As} is a stochastic innovation, drawn from a Normal distribution with standard deviation σ_A, that is common to productivity in sectors M and N (i.e., sector-neutral).

Competitive final producers repackage the intermediate inputs to produce consumption and investment goods. Consumption producers minimize the cost of producing a desired level of consumption goods, split between private consumption C_s and government consumption G_{Cs}, by solving the following Lagrangian problem:

$$\min_{Y_{MCs},Y_{NCs},P_{Cs}} P_{Ns}Y_{NCs} + P_{Ms}Y_{MCs} - P_{Cs}\left[Y_{NCs}^{\alpha_{NC}}Y_{MCs}^{1-\alpha_{NC}} - (C_s + G_{Cs})\right], \tag{10}$$

where α_{NC} governs the intensity of N-sector goods in the production of final consumption goods. In turn, government consumption follows a simple auto-regressive process:

$$G_{Cs} = \rho_{GC}G_{Cs} + \epsilon_{GCs}, \tag{11}$$

where the parameter ρ_{GC} governs the persistence of the shock process, and where ϵ_{GCs} is a stochastic innovation drawn from Normal distribution with standard deviations σ_{GC}. Investment producers solve the analogous problem:

$$\min_{Y_{MIs}, Y_{NIs}, P_{Is}} P_{Ms} Y_{MIs} + P_{Ns} Y_{MIs} - P_{Is} \left[Y_{NIs}^{\alpha_{NI}} Y_{MIs}^{1-\alpha_{NI}} - I_s \right], \tag{12}$$

with α_{NI} governing the intensity of N-sector goods in the production of final investment goods.

In addition to satisfying the first-order conditions for the optimization problems of households and firms, an equilibrium in the model has no borrowing (i.e., $B_s = 0 \;\; \forall s$) and is such that all factor markets and product markets clear. Accordingly, $Y_{Ms} = Y_{MCs} + Y_{MIs}$, $Y_{Ns} = Y_{NCs} + Y_{NIs}$, and $L_s = L_{Ms} + L_{Ns}$.

3.2. The IST Model

The IST model differs from the MFP model along two dimensions: 1) complete specialization in the assembly functions for final consumption and investment goods, which can be implemented as a parametric restriction on $\alpha_{NC} = 1$ and $\alpha_{NI} = 0$; and 2) capital predetermined at the aggregate rather than at the sectoral level. This second dimension changes the optimization problem in **??** above into:

$$\max_{Cs, I_s, K_{Ns}, K_{Ms}, K_{s+1}, B_s} E_t \sum_{s=t}^{\infty} \left[\tilde{\beta}_s \left((1-\eta) \log(C_s - \eta \bar{C}_{s-1}) - \frac{\chi_0}{1+\chi} V_s \left(L_s \right)^{1+\chi} \right) \right]. \tag{13}$$

Notice that households now choose K_{Ms} and K_{Ns} instead of K_{Ms+1} and K_{Ns+1}. Furthermore, Equation 5 is replaced by the following constraints:

$$K_{s+1} = (1-\delta_M) K_{Ms} + (1-\delta_N) K_{Ns} + I_s - \frac{\nu}{2} I_s \left(\frac{I_s}{I_{s-1}} - 1 \right)^2, \tag{14}$$

$$K_{Ms} + K_{Ns} = K_s. \tag{15}$$

4. Proving that the MFP Model is Consistent with Fisher's Long-Run Identification Scheme

In this section we prove analytically that the model described in Section 3.1 satisfies the restrictions imposed by the identification scheme in Fisher (2006) despite its multi-sector structure with different factor intensities across sectors.

Theorem 1. *In the long run, equiproportionate shocks to technology in the two production sectors M and N affect aggregate labor productivity but do not affect relative prices. Furthermore, shocks to technology in one production sector affect both aggregate labor productivity and relative prices.*

The proof to this theorem is given in two parts below and relies on the steady-state conditions in Table 2, which apply both to the MFP and the IST models. A corollary of this theorem is that the two-sector model of Section 3.1 can be used to interpret the permanent shocks to the relative price of investment and to labor productivity identified in Section 2. Since the steady-state restrictions of the MFP model, subject to appropriate parametric restrictions, also apply to the IST model in Section 3.2, the theorem applies to this second model, too. Notice also that since the proof relies on steady-state restrictions, the arguments below generalize to models with additional short run-features that leave the steady-state considerations unchanged. For instance, alternative utility functions also consistent with a balanced growth path, as described in King, Plosser, and Rebelo (1988), would leave the arguments below unchanged.

Table 2: Steady-State Restrictions

i)	$\beta\frac{R_N}{P_C C} - \frac{P_I}{P_C C} + \beta\frac{P_I}{P_C C}(1-\delta_N) = 0$	ii)	$\frac{R_N}{R_M} = \frac{1-\beta(1-\delta_N)}{1-\beta(1-\delta_M)}$
iii)	$R_M = P_M \alpha_M \frac{Y_M}{K_M}$	iv)	$W = P_M(1-\alpha_M)\frac{Y_M}{L_M}$
v)	$R_N = P_N \alpha_N \frac{Y_N}{K_N}$	vi)	$W = P_N(1-\alpha_N)\frac{Y_N}{L_N}$
vii)	$Y_M = K_M^{\alpha_M}(A_M L_M)^{1-\alpha_M}$	viii)	$Y_N = K_N^{\alpha_N}(A_N L_N)^{1-\alpha_N}$
ix)	$Y_{NC} = \alpha_{NC} C \frac{P_C}{P_N}$	x)	$Y_{NI} = \alpha_{NI} I \frac{P_I}{P_N}$
xi)	$C = Y_{NC}^{\alpha_{NC}} Y_{MCs}^{1-\alpha_{NC}}$	xii)	$I = Y_{NI}^{\alpha_{NI}} Y_{MI}^{1-\alpha_{NI}}$
xiii)	$P_I = \left(\frac{P_N}{\alpha_{NI}}\right)^{\alpha_{NI}} \left(\frac{P_{Ms}}{1-\alpha_{NI}}\right)^{1-\alpha_{NI}}$	xiv)	$P_{Cs} = \left(\frac{P_{Ns}}{\alpha_{NC}}\right)^{\alpha_{NC}} \left(\frac{P_M}{1-\alpha_{NC}}\right)^{1-\alpha_{NC}}$
xv)	$Y_M = Y_{MC} + Y_{MI}$	xvi)	$Y_N = Y_{NC} + Y_{NI}.$
xvii)	$L_M + L_N = L$	xviii)	$\delta_M K_M + \delta_N K_N = I$
xix)	$P_N = 1$		

4.1. The Long-Run Response of Relative Prices

Some quick preliminary manipulations are in order. Notice that the rental rates for the two types of capital are related to each other as shown in ii) in Table 2, so i) together with xiii), xiv) and xix) imply that

$$R_N = P_I\left(\frac{1}{\beta} - (1-\delta_N)\right), \tag{16}$$

$$R_M = P_I \left(\frac{1}{\beta} - (1 - \delta_M) \right). \tag{17}$$

Next, from iii) and vii), and from v) and viii) in Table 2, one can relate labor productivity at the sectoral level to the ratio of the sectoral price and the sectoral rate of return for capital:

$$\frac{Y_M}{L_M} = A_M \left(\alpha_M \frac{P_M}{R_M} \right)^{\frac{\alpha_M}{1-\alpha_M}}, \tag{18}$$

$$\frac{Y_N}{L_N} = A_N \left(\alpha_N \frac{P_N}{R_N} \right)^{\frac{\alpha_N}{1-\alpha_N}}. \tag{19}$$

In turn, the relative price of intermediate outputs can be related sectoral labor productivity measures using equations iv) and vi) in Table 2, yielding

$$\frac{P_M}{P_N} = \frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{Y_N}{L_N} \frac{L_M}{Y_M}. \tag{20}$$

And combining equations 17, 16, 18, and 19 with Equation 20, one can see that

$$\frac{P_M}{P_N} = \left(\psi \frac{A_N}{A_M} \right)^{\frac{(1-\alpha_N)(1-\alpha_M)}{(1-\alpha_N)(1-\alpha_M)+(1-\alpha_M)(1-\alpha_{NI})\alpha_N+(1-\alpha_N)\alpha_{NI}\alpha_M}}, \text{ where} \tag{21}$$

$$\psi = \left(\frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{\left(\alpha_N \frac{1}{\left(\frac{1}{\alpha_{NI}}\right)^{\alpha_{NI}} \left(\frac{1}{1-\alpha_{NI}}\right)^{1-\alpha_{NI}} \left(\frac{1}{\beta}-(1-\delta_N)\right)} \right)^{\frac{\alpha_N}{1-\alpha_N}}}{\left(\alpha_M \frac{1}{\left(\frac{1}{\alpha_{NI}}\right)^{\alpha_{NI}} \left(\frac{1}{1-\alpha_{NI}}\right)^{1-\alpha_{NI}} \left(\frac{1}{\beta}-(1-\delta_M)\right)} \right)^{\frac{\alpha_M}{1-\alpha_M}}} \right).$$

Equation 21 implies that changes in technology in a single production sector will affect relative prices, but equiproportionate changes in technology in the two production sectors, dubbed neutral MFP shocks for the VAR of Section 2, will not affect relative prices owing to wage equalization associated with long-run labor mobility. This result applies to the IST model, too, but in that case, complete specialization in the production of final goods implies that relative prices can be expressed more simply as

$$\frac{P_M}{P_N} = \left(\psi_{IST} \frac{A_N}{A_M} \right)^{1-\alpha_N}, \quad \text{where } \psi_{IST} = \left(\frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{\left(\alpha_N \frac{1}{\frac{1}{\beta}(1-\beta(1-\delta_N))} \right)^{\frac{\alpha_N}{1-\alpha_N}}}{\left(\alpha_M \frac{1}{\frac{1}{\beta}(1-\beta(1-\delta_M))} \right)^{\frac{\alpha_M}{1-\alpha_M}}} \right). \tag{22}$$

4.2. The Long-Run Response of Labor Productivity

Define aggregate labor productivity (at constant prices) as:

$$\frac{Y_{Mt}+Y_{Nt}}{L} = \frac{Y_{Mt}}{L_{Mt}}\frac{L_{Mt}}{L} + \frac{Y_{Nt}}{L_{Nt}}\frac{L_{Nt}}{L} \tag{23}$$

First work on relating $\frac{L_{Mt}}{L}$ and $\frac{L_{Nt}}{L}$ to the conditions for an equilibrium in Table 2. Using iv), iv), and xvii), one can obtain

$$\frac{L_M}{L} = \frac{(1-\alpha_M)\frac{P_M}{P_N}}{(1-\alpha_M)\frac{P_M}{P_N}+(1-\alpha_N)\frac{Y_N}{Y_M}}, \tag{24}$$

$$\frac{L_N}{L} = \frac{(1-\alpha_N)\frac{Y_N}{Y_M}}{(1-\alpha_M)\frac{P_M}{P_N}+(1-\alpha_N)\frac{Y_N}{Y_M}}. \tag{25}$$

Through equation 21, one can see that the term $\frac{P_M}{P_N}$ in equations 24 and 25, can be expressed as a function of technology levels across sectors and parameters. In an appendix, we show that the same is true for the term $\frac{Y_M}{Y_N}$. Next, using the equations in Table 2, one can express $\frac{Y_M}{L_M}$ and $\frac{Y_N}{L_N}$ as:

$$\frac{Y_M}{L_M} = A_M\left(\frac{\alpha_M}{\left(\frac{1}{\beta}-(1-\delta_M)\right)}\frac{P_M}{P_I}\right)^{\frac{\alpha_M}{1-\alpha_M}} \tag{26}$$

$$\frac{Y_N}{L_N} = A_N\left(\frac{\alpha_N}{\left(\frac{1}{\beta}-(1-\delta_N)\right)}\frac{P_N}{P_I}\right)^{\frac{\alpha_N}{1-\alpha_N}} \tag{27}$$

Notice that, using Equation xiii) in Table 2, one can express $\frac{P_M}{P_I}$ and $\frac{P_N}{P_I}$ in equations 26 and 27 in terms of $\frac{P_M}{P_N}$ and parameters. In turn, through equation 21, one can see that $\frac{P_M}{P_N}$ is a function of technology levels across sectors and parameters.

Accordingly, each of the terms in Equation 23, is a function of parameters and of the levels of multi-factor productivity in sectors M and N. Accordingly, due to long-run perfect mobility of labor, labor productivity will vary permanently both in response to sectoral MFP shocks that affect the relative level of A_M and A_N, and in response to neutral MFP shocks that affect the levels of A_M and A_N equiproportionately.

Notice that these results also apply to the IST model, but, with complete specialization in the assembly of final goods assumed in that model, labor productivity simplifies to:

$$\begin{aligned}\frac{Y_M+Y_N}{L} &= \frac{Y_M}{L_M}\frac{L_M}{L} + \frac{Y_N}{L_N}\frac{L_N}{L} \\ &= A_M\left(\frac{\alpha_M}{(1-\beta(1-\delta_M))}\right)^{\frac{\alpha_M}{1-\alpha_M}}\frac{(1-\alpha_M)}{(1-\alpha_M)+(1-\alpha_N)\phi}\end{aligned} \tag{28}$$

$$+A_M^{\alpha_N} A_N^{1-\alpha_N} \left(\frac{\alpha_N}{\psi_{IST}^{1-\alpha_N} \left(1-\beta(1-\delta_N)\right)} \right)^{\frac{\alpha_N}{1-\alpha_N}} \frac{(1-\alpha_N)\phi}{(1-\alpha_M)+(1-\alpha_N)\phi},$$

where $\phi = \left(\frac{1-\delta_M \frac{\alpha_M}{(1-\beta(1-\delta_M))}}{\delta_N \frac{\alpha_N}{(1-\beta(1-\delta_N))}} \right)$.

In sum, based on the results shown in this section, and on Equation 21, our baseline model is consistent with the scheme in Fisher (2006). [11]

Figure 3 offers a numerical confirmation of our analytical proof. It shows the response of the relative price of investment and of labor productivity to all the shocks included in the model. Among the shocks included in the model, the only shock that affects the price of investment permanently is an MFP shock in the machinery sector. Moreover, the only two shocks that affect the level of labor productivity permanently are the MFP shock in the machinery sector and the neutral MFP shock (constructed as MFP shocks in both sectors).

5. Discriminating Across Models Based on the VAR Results

Having established that the identification scheme for the VAR estimates is consistent with both variants of our richer model, we proceed by comparing model and VAR estimates. One approach typically used to discriminate across models based on VAR evidence is to check whether the model response to a certain shock is consistent or not with the empirical evidence from the VAR.[12] For our purposes, the problem with this approach is that the VAR confidence intervals for standard significance levels are so wide, as noted above in the description of Figure 1, that we would not be able to tell the models apart.

As noted in Erceg, Guerrieri, and Gust (2005), even imprecise tools such as our VAR can still be useful in discriminating across models. For instance, taking one of the models as the data-generating process, one could check if the VAR implies a bias in the point estimates of the impulse response functions in a certain direction. If that bias is reversed under the alternative model, then even an imprecise tool can offer sharp discriminating evidence. To investigate this possibility, we estimated the same VAR and used the same identification scheme to construct the impulse response functions in Figure 1 based on data generated from the two alternative DSGE models.

[11] Notice that Fisher (2006) defined aggregate labor productivity in terms of consumption units, i.e., $\frac{Y_{Mt}}{L_{Mt}} \frac{L_{Mt}}{L} \frac{P_M}{P_N} + \frac{Y_N}{L_N} \frac{L_N}{L}$ using our notation, rather than at constant prices. Even under that alternative aggregation, labor productivity is affected both by equiproportionate shocks across production sectors and by shocks to a single production sector.

[12] See, for instance, Gali (1999) and Gali and Rabanal (2004).

5.1. Calibration

For each model, we employ a mix of calibration and estimation. The estimated parameters include the autoregressive coefficients and the standard deviations for all the shock processes. We describe first the calibrated parameters for each model.

5.1.1. Calibrated Parameters for the MFP Model

All calibrated parameters for the MFP model are reported in Table 3. The discount factor β is set at 0.99, implying a steady state short-term real interest rate equal to about 4 percent on an annualized basis. The parameter χ is set to 1/1000, implying that the functional for labor disutility is almost linear in the labor input.

The machinery sector of our model has two components. The first component is the NIPA definition of "equipment and software" investment, after excluding the transportation, wholesale, and retail margins from the Input-Output Tables. Most of the industries whose output is used in equipment and software produce exclusively for equipment and software. The second component of our machinery sector comprises those inputs for consumption assembly from all the industries that produce inputs used in both the NIPA definition of equipment and software investment and of consumption. These IO Table industries are: (334) Computer and Electronic Products; (335) Electrical Equipment, Appliances, and Components; (513) Broadcasting and Telecommunications; (514) Information and Data Processing Services; and (5412OP) Miscellaneous Professional, Scientific and Technical Services. Reflecting this choice of sectors, We combined data for the net capital stock of private nonresidential fixed assets from the U.S. Bureau of Economic Analysis, with data from the Input-Output Bridge Table for Private Equipment and Software. The first data set contains data on the size of equipment and non-equipment capital stocks by sector. The second data set allowed us to ascertain the commodity composition of private equipment and software. Finally, we used BEA data to establish a sector's value added output. We focused on the year 2004, but similar sector-specific production functions would be implied by different vintages of data.

Our calculations show that the machinery-producing sector is more intensive in capital than the aggregate economy (the share of capital in production is $\alpha_M^S = 0.54$) and, accordingly, the larger non-machinery sector is less intensive (the share of capital in production is $\alpha_N = 0.28$).

The model captures the commingling implied by the bridge tables through assembly functions that specify how inputs from the M and N sectors are combined to obtain consumption and investment. The share parameters for the assembly functions are set as follows: $\alpha_{NI} = 0.42$, implying that non-machinery goods account for 42 percent of investment; $\alpha_{NC} = 0.96$, implying that non-machinery goods account for 96 percent of consumption.

Based on analogous sectoral definitions and data, we set the depreciation rate in the machinery sector, δ_M at a slightly higher 0.027 than the rate δ_N for the non-machinery sector, set at 0.023.

The parameter governing consumption habits η is 0.95 and the parameter ν governing investment adjustment costs is 50. These two parameter choices allow a match between the population estimate of the correlation between consumption and investment at business cycle frequencies conditional on (permanent) shocks to machinery sector technology A_M and the mode of the same moment from the VAR (conditional on permanent shocks to the relative price of investment), equal to 0.95.

5.1.2. Calibrated Parameters for the IST Model

All calibrated parameters for the IST model are reported in Table 4. We emphasize here the parameters that vary relative to those presented for the MFP model.

The parameters governing the assembly functions are set so that there is complete specialization: consumption and structures investment are assembled using inputs from the N sector only, while equipment investment is assembled using inputs from the M sector only. Accordingly, $\alpha_{NI} = 0$ and $\alpha_{NC} = 1$. The depreciation rates, δ_M and δ_N, are both set to 0.025.

Table 3: Calibrated Parameters for the MFP Model

Parameter	Determines	Parameter	Determines
Utility Function			
$\beta = 0.99$	Discount factor	$\eta = 0.95$	Consumption habits
$\chi = 1/1000$	Labor-supply elasticity $= 1/\chi$	$\chi_0 = 0.88$	Steady-state hours worked = 1
Intermediate Goods Production			
$\alpha_M = 0.54$	Capital share in the M sector	$\alpha_N = 0.28$	Capital share in the N sector
Depreciation Rates			
$\delta_M = 0.027$	Machinery sector	$\delta_N = 0.023$	Non-machinery sector
Consumption and Investment Assembly			
$\alpha_{NI} = 0.42$	N goods intensity for investment	$\alpha_{NC} = 0.96$	N goods intensity for consumption
$\nu = 50$	Investment adjustment costs		

Table 4: Calibrated Parameters for the IST Model

Parameter	Determines	Parameter	Determines
Utility Function			
$\beta = 0.99$	Discount factor	$\eta = 0.95$	Consumption habits
$\chi = 1/1000$	Labor-supply elasticity $= 1/\chi$	$\chi_0 = 0.89$	Steady-state hours worked = 1
Intermediate Goods Production			
$\alpha_M = 0.3$	Capital share in the M sector	$\alpha_N = 0.3$	Capital share in the N sector
Depreciation Rates			
$\delta_M = 0.025$	Machinery sector	$\delta_N = 0.025$	Non-machinery sector
Consumption and Investment Assembly			
$\alpha_{NI} = 0$	N goods intensity for investment	$\alpha_{NC} = 1$	N goods intensity for consumption
$\nu = 50$	Investment adjustment costs		

5.1.3. Estimated Parameters for the MFP and IST Models

For the estimation, we focus on matching the variance, the covariance, and the first autocorrelation of the same five variables used in the VAR: the growth rate of the relative price of investment, labor productivity growth, hours per capita, the growth rate of equipment and software per capita, and the growth rate of consumption per capita. To weigh the various moments we use the diagonal of the simulated method of moments weighting matrix. We estimate the parameters governing the shock processes (labor supply, consumption, and government spending shocks).

We read out the standard deviations for the innovations for the neutral MFP and sectoral MFP or IST shocks from the VAR estimates. The standard deviation of the neutral MFP shock is chosen to match the VAR long-run response of labor productivity to a one-standard-deviation MFP shock. The standard deviation of the sectoral MFP or IST shocks is chosen to match the VAR long-run response of the relative price of investment to a one-standard-deviation shock to the relative price of investment. Under the calibration for the aggregate model, sectoral MFP shocks and IST shocks are equivalent and we drop the sectoral MFP shocks. Under the calibration that maintains the sectoral detail, we drop the IST shocks. The estimation results are summarized in Tables 5 and 6.

Table 5: Parameters Governing the Shocks in the MFP Model

Parameter	Determines	Parameter	Determines
Standard Deviations of Shocks			
$\sigma_M = 0.0508$	M-sector productivity	$\sigma_A = 0.0037$	Neutral TFP
$\sigma_V = 0.1198$	Labor supply	$\sigma_{GC} = 0.0319$	Government consumption
$\sigma_\beta = 0.0002$	Risk premium		
Autoregressive Coefficient of Shocks			
$\rho_V = 0.26$	Labor supply	$\rho_{GC} = 0.95$	Government consumption
$\rho_\beta = 0$	Risk premium		

5.2. Monte Carlo Results

For this experiment, we used 1000 randomly drawn samples of the same length as the baseline sample. We found that the differential implications of the two alternative models regarding the responses to MFP and IST shocks are swamped by the uncertainty associated with a VAR estimated with long-run restrictions on short sample, and still do not allow us to tell the models apart. The results for

Table 6: Parameters Governing the Shocks in the IST model

Parameter	Determines	Parameter	Determines
Standard Deviations of Shocks			
$\sigma_M = 0.0411$	M-sector productivity	$\sigma_A = 0.0037$	Neutral TFP
$\sigma_V = 0.1369$	Labor supply	$\sigma_{GC} = 0.0645$	Government consumption
$\sigma_\beta = 0.0645$	Risk premium		
Autoregressive Coefficient of Shocks			
$\rho_V = 0$	Labor supply	$\rho_{GC} = 0.95$	Government consumption
$\rho_\beta = 0$	Risk premium		

this experiment are reported in Appendix B.[13] The appendix also shows that when the estimation sample is extended to include 10,000 observations, the mass of the VAR responses hugs the contours of the model responses, indicating that the VAR can recover the shocks used in the data-generating process.[14]

While the estimated impulse response functions do not offer evidence sufficient to discriminate between the two models for the empirically-relevant sample size, a key difference between the two models is the correlation between consumption and investment at business cycle frequencies, conditional on shocks to the price of investment.

The population estimate for this correlation equals a modest 0.17 for the aggregate model with IST shocks and 0.94 for the two-sector model with MFP shocks (which, by construction, matches the mode of the distribution for the analogous estimate from the VAR). The vertical lines in Figure 4 show these two correlations. For convenience, the red shaded area reproduces, in the top left panel, the PDF of the analogous correlation based on the VAR. The top right panel shows the CDF for the same estimate. These distribution indicate that the correlations in the range implied by the aggregate model would be unlikely realizations, pointing to the two-sector model as the more plausible candidate to explain the comovement properties extracted from the observed U.S. data.

In addition to the distributions from the VAR, Figure 4 also reports distributions (PDFs on the left CDFs on the right) for the correlation between consumption and investment, obtained through the same Monte Carlo experiment described above for the impulse response functions. These distributions allow us to gauge how sampling uncertainty affects the estimates for the correlation

[13] See figures 6 and 7.

[14] The ABCD test of Fernndez-Villaverde, Rubio-Ramrez, Sargent, and Watson (2007) is inconclusive for our models - the relevant eigenvalue for the ABCD test is one reflecting unit roots in the shock processes.

between consumption and investment when each of the alternative models is taken to be the data-generating process. The solid line shows the distributions for the sectoral MFP model. The dashed line shows the distributions for the aggregate IST model. As for the case of the impulse response functions, the distributions indicate that the VAR is an imprecise tool with substantial mass for the density function away from the pseudo-true values for each of the two models. Parsing out the sources for this imprecision for the specific models at hand, as done by Erceg, Guerrieri, and Gust (2005), is beyond the scope of this paper. Nonetheless, it is worthwhile to note that the CDF for the two-sector model is uniformly closer to the CDF for the VAR estimated on observed U.S. data, pointing again to the two-sector model as a more plausible data-generating process.

The tables are turned when it comes to neutral MFP shocks (shocks that move labor productivity permanently but not relative prices) which are the focus of the lower panels in Figure 4. For the baseline sample size, the distributions for the estimates correlation between consumption and investment are quite dispered for the VAR estimated on observed data. VARs estimated on data drawn from our two models also show substantial uncertainty for this moment. Nonetheless, the Monte Carlo results point to the aggregate IST model as yielding a closer match to the data in this respect.

One way to weigh these two opposite results for sectoral and neutral productivity shocks is by the relative importance of each shock in driving business cycle fluctuations. As the VAR evidence points to the sectoral shocks as most prominent, as shown earlier in Table 1, on balance, we still view the sectoral IST model as providing a better match to the data. Of course, in addition, the sectoral MFP model also has the virtue of reflecting important features of the production structure captured by the U.S. Input-Output Tables.

6. Conclusion

Consumption and investment comove over the business cycle. Our estimates show that consumption and investment also comove conditional on shocks that change the price of investment permanently. Our finding obtains in our baseline sample, from 1982:Q3 to 2008:Q3, broadly coinciding with the Great Moderation, as well as in our full sample encompassing all publicly available data and spanning the period from 1948:Q2 through 2015:Q1.

We show that this comovement can be used to discriminate between alternative models of the business cycle. Heretofore, the set of models used to interpret permanent movements in the relative price of investment included one-sector models with IST shocks, or multi-sector models that could be aggregated to a one-sector model. We showed that, in fact, the set of admissible models also includes a two-sector model that cannot be aggregated. We found that this two-sector model matches more closely the evidence of a positive correlation between consumption and investment, conditional on shocks that move the price of investment permanently.

In this paper we have examined the connection between empirical evidence from movements in the relative price of investment with sectoral and aggregate treatments of multi-factor productivity changes using DSGE models. A fruitful avenue for further research would be to explore the relationship between sectoral MFP shocks inferred from identified VARs and sectoral measures of MFP levels obtained from growth accounting exercises in the tradition of Solow (1957) and Griliches and Jorgenson (1966). A related direction for further research would be to characterize the general class of DSGE models that is consistent with the restrictions implied by growth accounting exercises.

References

Basu, S., J. G. Fernald, J. Fisher, and M. Kimball (2013). Sector-Specific Technical Change. Mimeo, Federal Reserve Bank of San Francisco.

Christiano, L. J. and T. J. Fitzgerald (1998). The Business Cycle: It's Still a Puzzle. *Economic Perspectives, Federal Reserve Bank of Chicago*, 56–83. Federal Reserve Bank of Minneapolis: Staff Report 200.

Christiano, L. J., C. Ilut, R. Motto, and M. Rostagno (2008). Monetary Policy and Stock Market Boom-Bust Cycle. European Central Bank, Working Paper Series 955.

Cooley, T. F. and M. Dwyer (1998). Business cycle analysis without much theory: A look at structural VARs. *Journal of Econometrics 83*(1-2), 57–88.

Erceg, C. J., L. Guerrieri, and C. Gust (2005). Can Long-Run Restrictions Identify Technology Shocks? *Journal of the European Economic Association 3*(6), 1237–1278.

Faust, J. and E. M. Leeper (1997). When Do Long-Run Identifying Restrictions Give Reliable Results? *Journal of Business & Economic Statistics 15*(3), 345–53.

Fernndez-Villaverde, J., J. F. Rubio-Ramrez, T. J. Sargent, and M. W. Watson (2007). ABCs (and Ds) of Understanding VARs. *American Economic Review 97*(3), 1021–1026.

Fisher, J. (2006). The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Journal of Political Economy 114*(3), 413–452.

Gali, J. (1999). Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *American Economic Review 89*, 249–271.

Gali, J. and P. Rabanal (2004). Technology Shocks and Aggregate Fluctuations: How Well Does the RBC Model Fit Postwar U.S. Data? NBER Working Papers 10636, National Bureau of Economic Research, Inc.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-Run Implications of Investment-Specific Technological Change. *American Economic Review 87*, 342–362.

Greenwood, J., Z. Hercowitz, and P. Krusell (2000). The Role of Investment-Specific Technological Change in the Business Cycle. *European Economic Review 44*, 91–115.

Griliches, Z. and D. W. Jorgenson (1966). Sources of Measured Productivity Change: Capital Input. *The American Economic Review 56*(1/2), 50–61.

Guerrieri, L., D. Henderson, and J. Kim (2014). Modeling Investment-Sector Efficiency Shocks:

When Does Disaggregation Matter? *International Economic Review 55*, 891–917.

Jaimovich, N. and S. Rebelo (2009). Can News About the Future Drive the Business Cycle? *American Economic Review 99*(4), 1097–1118.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988). Production, growth and business cycles : I. The basic neoclassical model. *Journal of Monetary Economics 21*(2-3), 195–232.

Peersman, G. and R. Straub (2006). Putting the New Keynesian Model to a Test. Working Papers of Faculty of Economics and Business Administration, Ghent University, Belgium 06/375, Ghent University, Faculty of Economics and Business Administration.

Ramey, V. A. and M. D. Shapiro (1998). Displaced Capital. NBER Working Papers 6775, National Bureau of Economic Research, Inc.

Runkle, D. E. (1987). Vector Autoregressions and Reality. *Journal of Business and Economic Statistics 5*, 437–442.

Solow, R. (1957). Technical change and the aggregate production function. *Review of Economics and Statistics 39*(3), 312–320.

Figure 1: VAR Estimates of the Response to a One-Standard Deviation Shock that Lowers the Level of the Relative Price of Investment Permanently

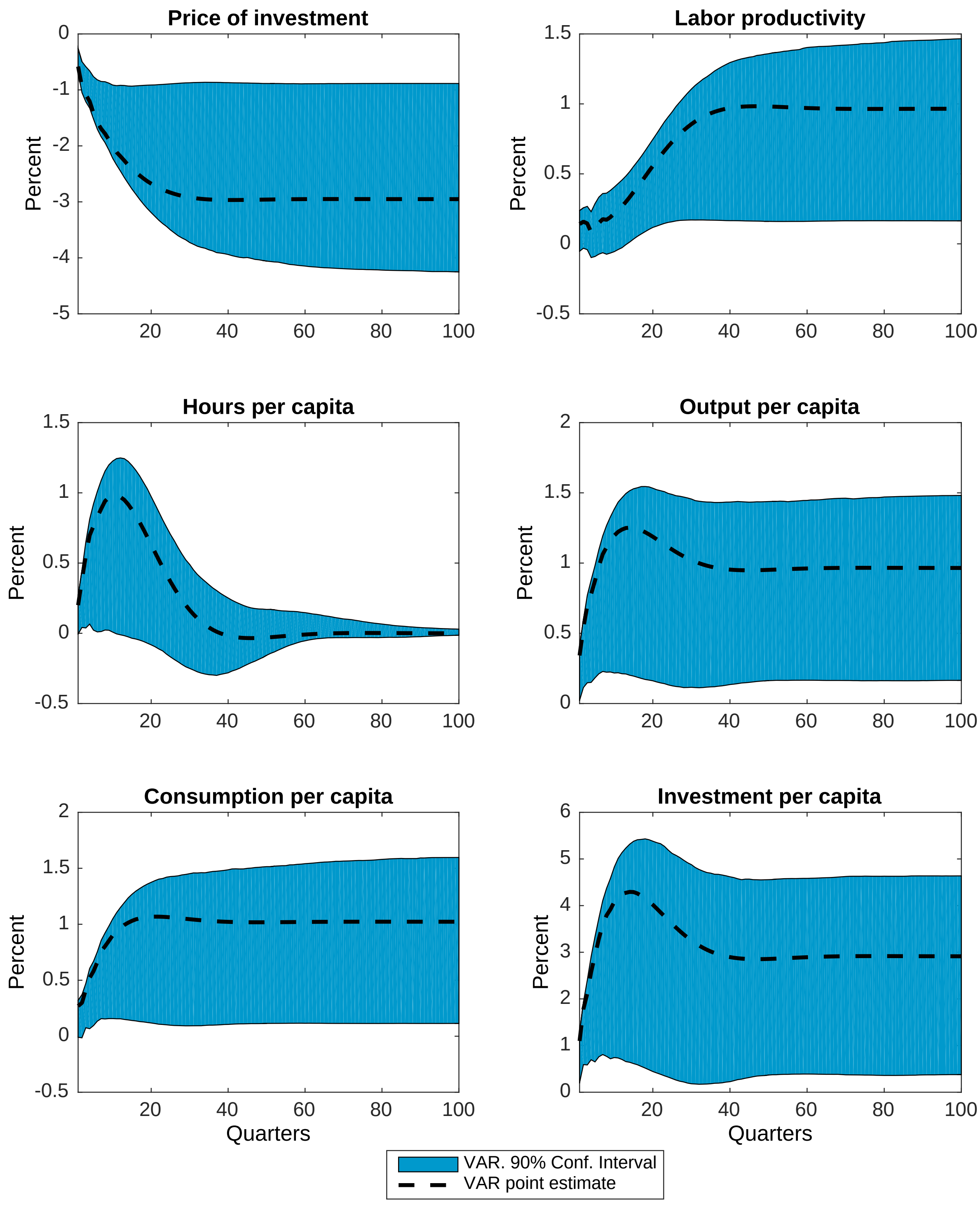


Figure 2: Cumulative Distribution Function for the Estimate of the Correlation between Investment and Consumption at Business Cycle Frequencies

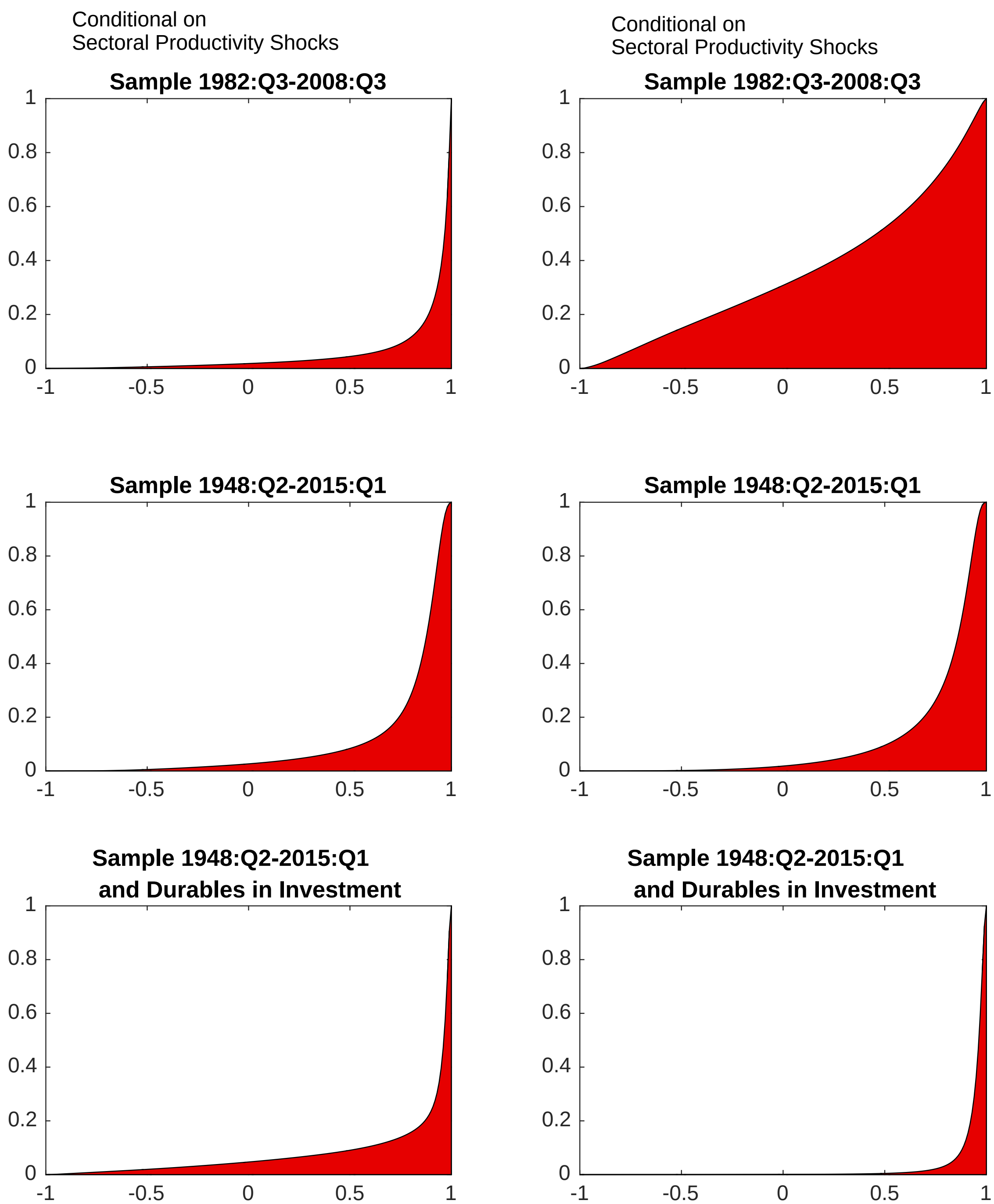
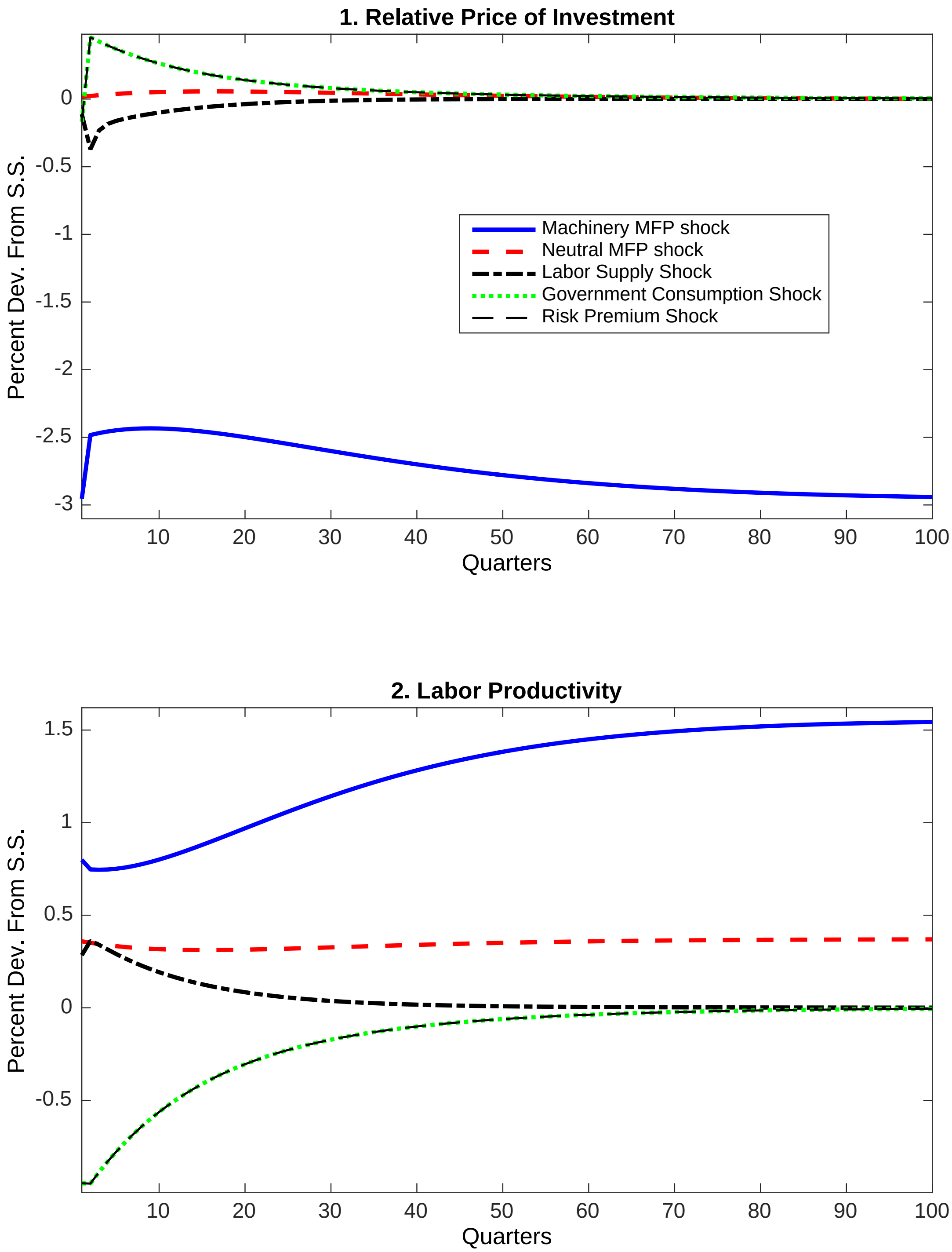
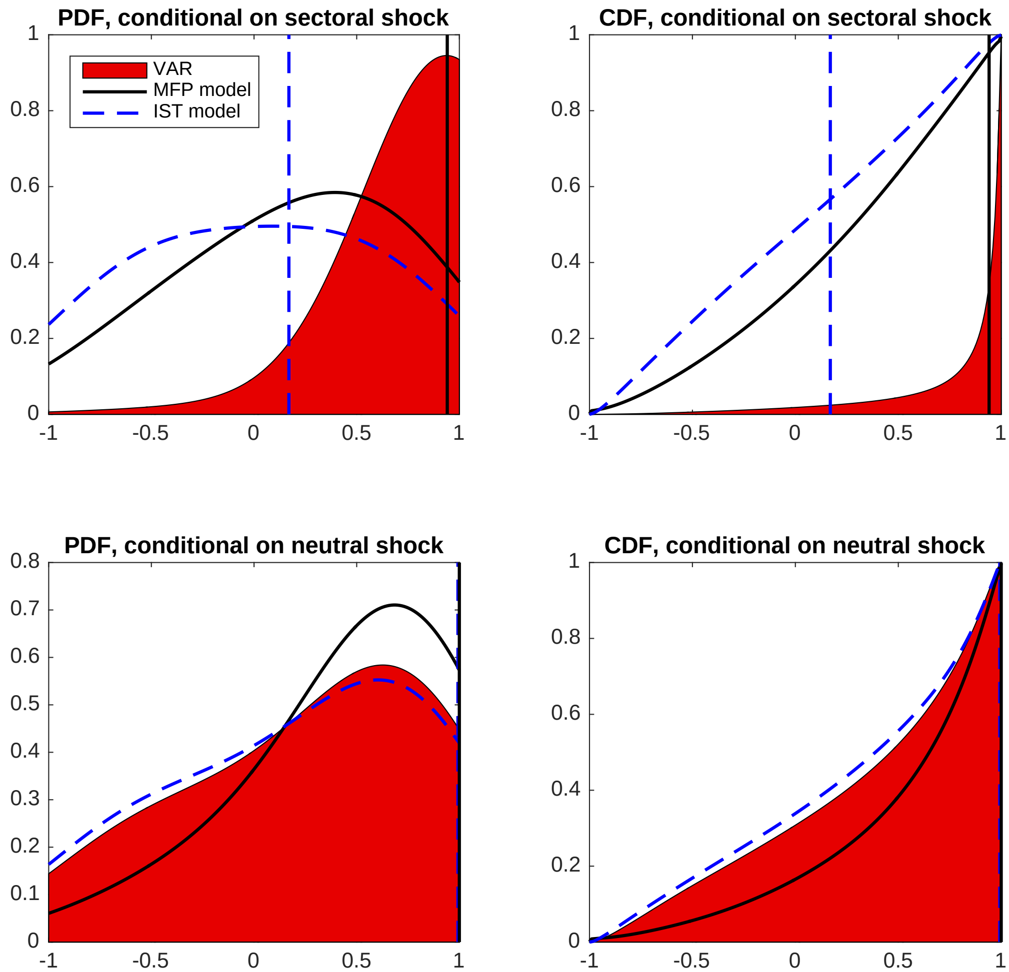


Figure 3: Properties of the Sectoral MFP Model: The Responses of the Relative Price of Investment and of Labor Productivity to Various Shocks



All the shocks considered are size at one standard deviation. Section 5.1.3 reports the estimation strategy for these parameters and their sizes.

Figure 4: Cumulative Distribution Function for the Estimate of the Correlation Between Consumption and Investment at Business Cycle Frequencies, Conditional on Shocks that Lower the Price of Investment Permanently: VAR and DSGE Model Results



For convenience, the shaded area reports again the CDF for estimates the correlation between consumption and investment conditional on shocks that move the price of investment permanently from a VAR for the baseline sample 1982:Q3-2008:Q3. The vertical lines denote population estimates conditional on shocks that move the relative price of investment permanently in the aggregate model with IST shocks and in the sectoral model with MFP shocks. The CDF denoted by a dashed line pertains to a Monte Carlo experiment, in which the VAR is estimated on data generated from the aggregate MFP model, as described in Section 5.The CDF denoted by a solid line pertains to a Monte Carlo experiment, in which the VAR is estimated on data generated from the sectoral IST model, also described in Section 5.

A. Appendix: Additional Results from the VAR

Section 2 provides a description of our VAR, identification strategy, and estimated responses to a shock that moves permanently the relative price of investment. For completeness, Figure 5 shows the estimates of the response from to a one standard deviation shock that increases permanently the level of labor productivity but that does not have a long-run effect on the level of the relative price of investment. Again, for the variables that overlap, our results are close to those in Fisher (2006).

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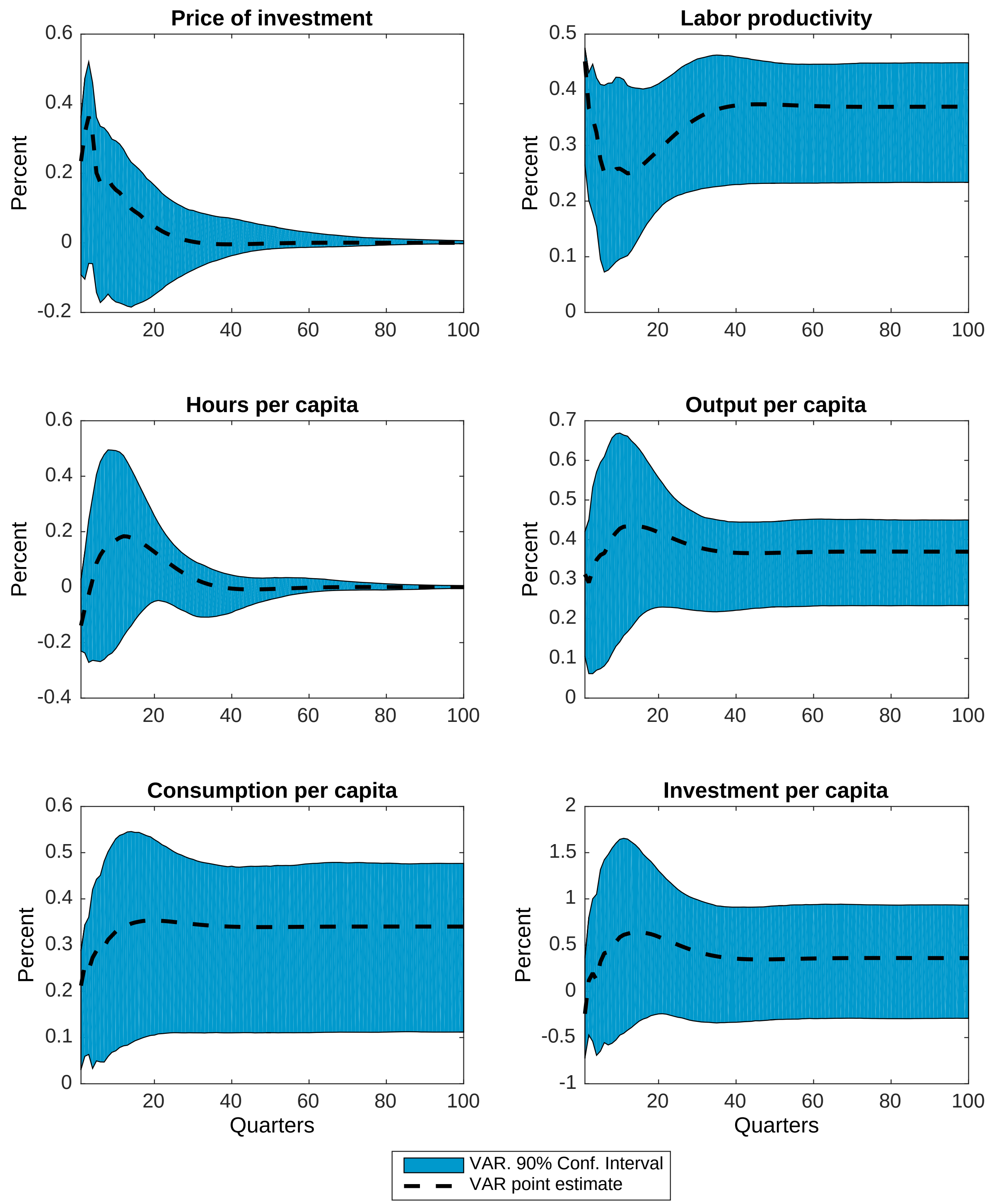
Figure 5: VAR Estimates of the Response to a One-Standard Deviation Shock that Increases the Level of Labor Productivity Permanently



B. Appendix: Additional Results of Monte Carlo Experiment

The red lines in Figure 6 show the responses to an MFP shock in the machinery sector of our two-sector model. By construction, the long-run response of the relative price of investment is normalized to match the response estimated from the VAR, but the short-run response is left unconstrained. The responses of the price of investment, consumption and investment from the model fall broadly within the 90% VAR confidence intervals (denoted by the dashed vertical lines), with the exception of some short run departures. The areas shaded in solid red show the results of a Monte Carlo experiment in which 1000 samples of the same length as the observed data were drawn using our two-sector model. For each sample we re-estimated the same VAR as for the observed data. The shaded areas are 90% confidence intervals for the response to a shock that lowers the relative price of investment permanently. There is substantial overlap between the areas shaded in solid red and those in dashed blue indicating that the VAR results could have been generated from a random sample from our two-sector model.

Similar considerations apply to analogous results for the IST shock shown in Figure 7 and to the responses to a neutral shock for the MFP model (Figure 8) and for the IST model (Figure 9). Accordingly, it would be hard disentangle the IST and MFP models based on this evidence.

Figure 10 shows that when we posit a large Monte Carlo sample of 10,000 observations, the VAR recovers estimates of the sectoral MFP shock that closely match the pseudo-true responses in the MFP model (in this case, we extended the VAR to include 50 lags). Finally, Figure 11 shows analogous results for the IST shock in the aggregate IST model.

Figure 6: The VAR Response to a One-Standard Deviation Shock that Lowers the Relative Price of Investment Permanently, Compared Against the Response to an MFP shock in the Machinery Sector of the Two-Sector Model and Against VAR Estimates Based on a Monte Carlo Experiment

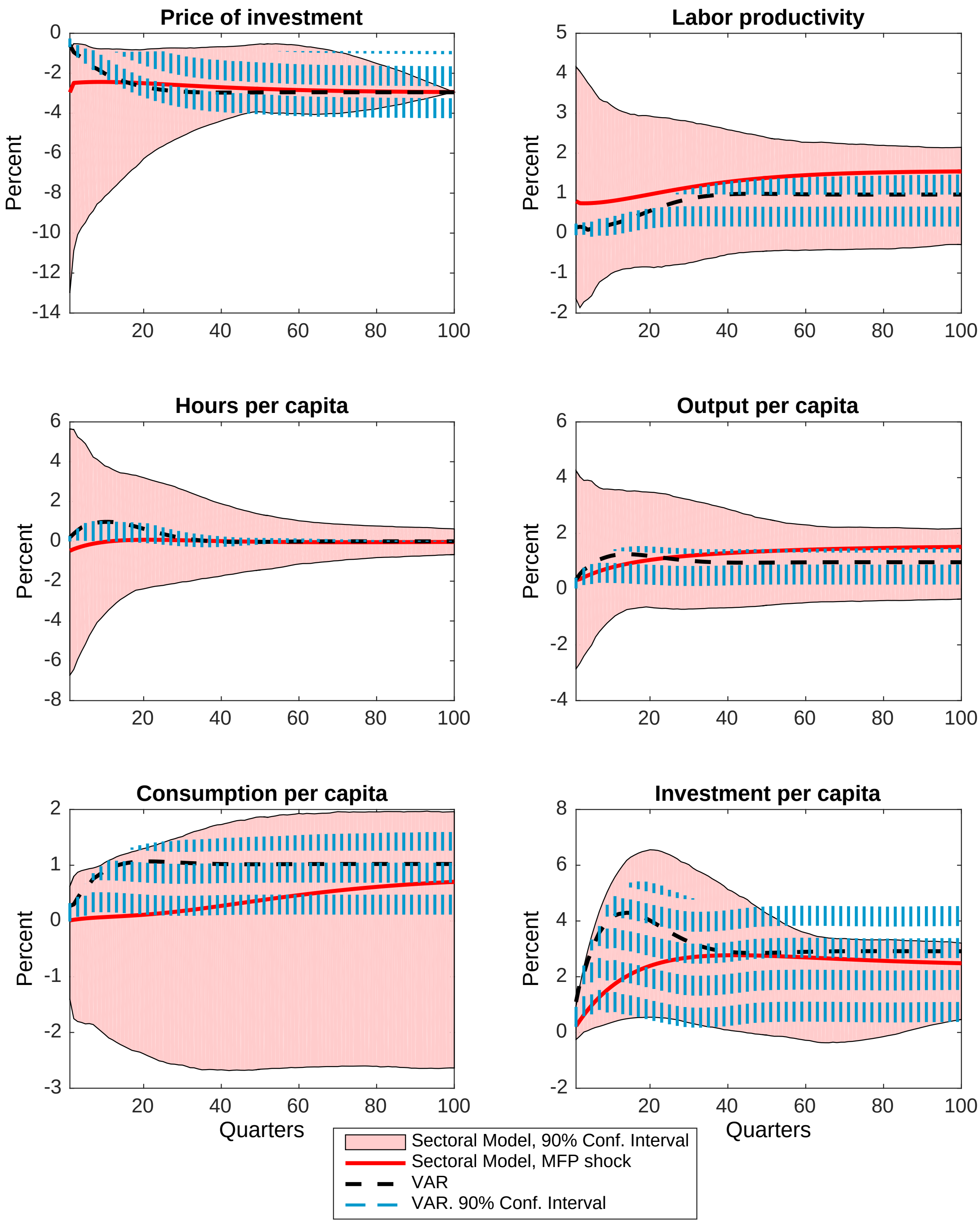


Figure 7: The VAR Response to a One-Standard Deviation Shock that Lowers the Relative Price of Investment Permanently, Compared Against the Response to an IST shock in the Aggregate Model and Against VAR Estimates Based on a Monte Carlo Experiment

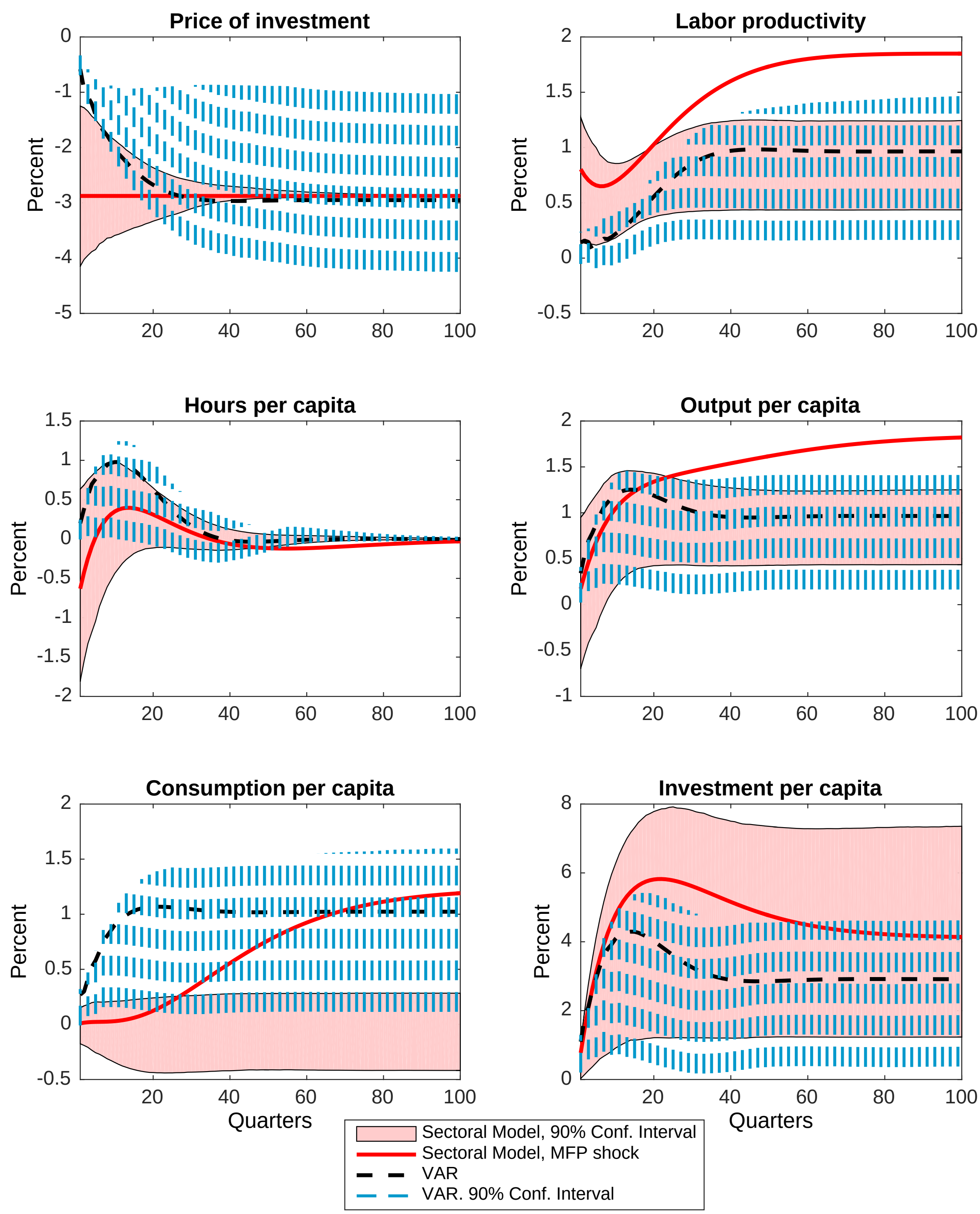


Figure 8: The VAR Response to a One-Standard Deviation Shock that Raises the Level of Labor Productivity Permanently, Compared Against the Response to a neutral Technology shock in the Two-Sector Model and Against VAR Estimates Based on a Monte Carlo Experiment

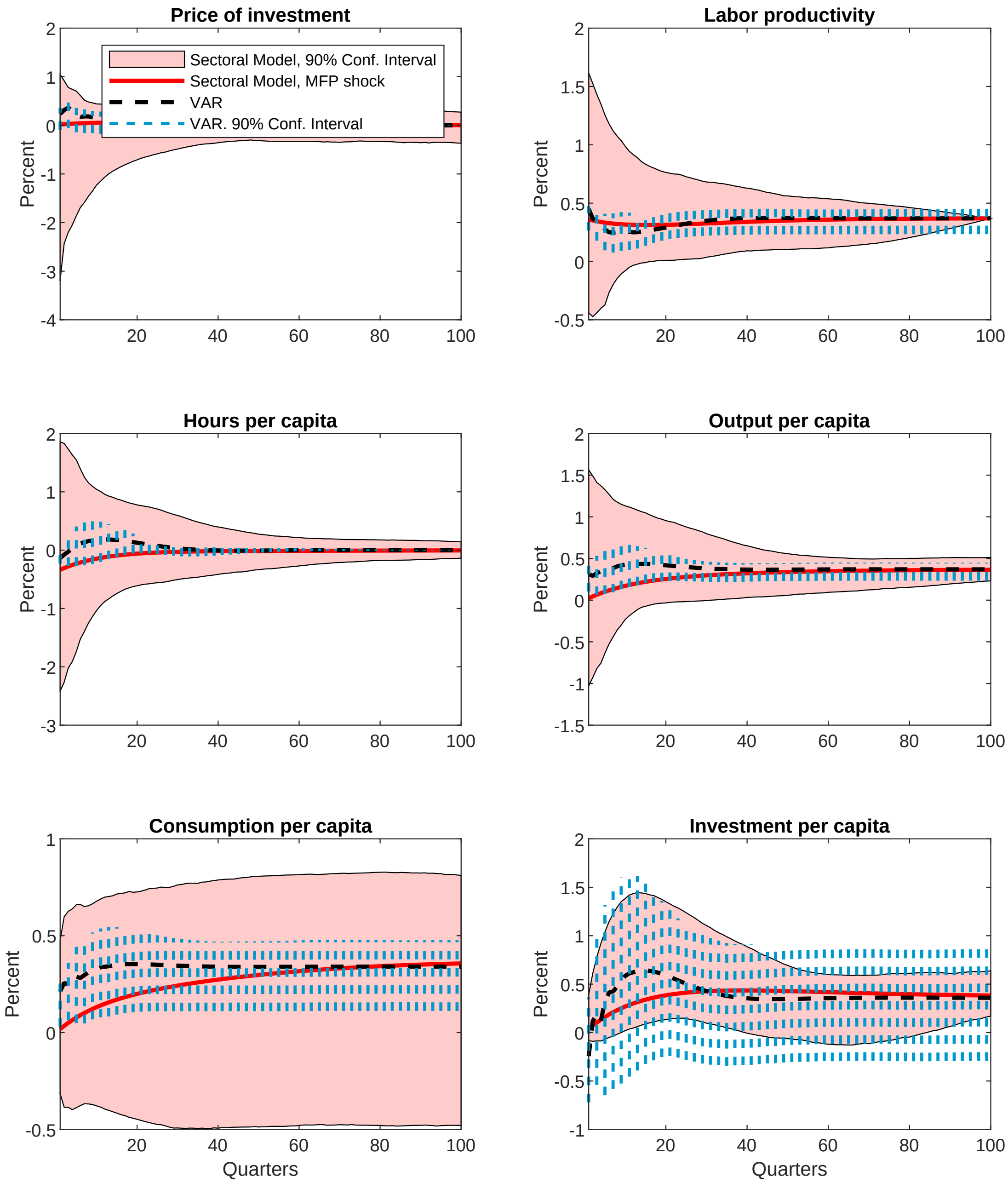


Figure 9: The VAR Response to a One-Standard Deviation Shock that Raises the Level of Labor Productivity Permanently, Compared Against the Response to a neutral Technology Shock in the Aggregate Model and Against VAR Estimates Based on a Monte Carlo Experiment

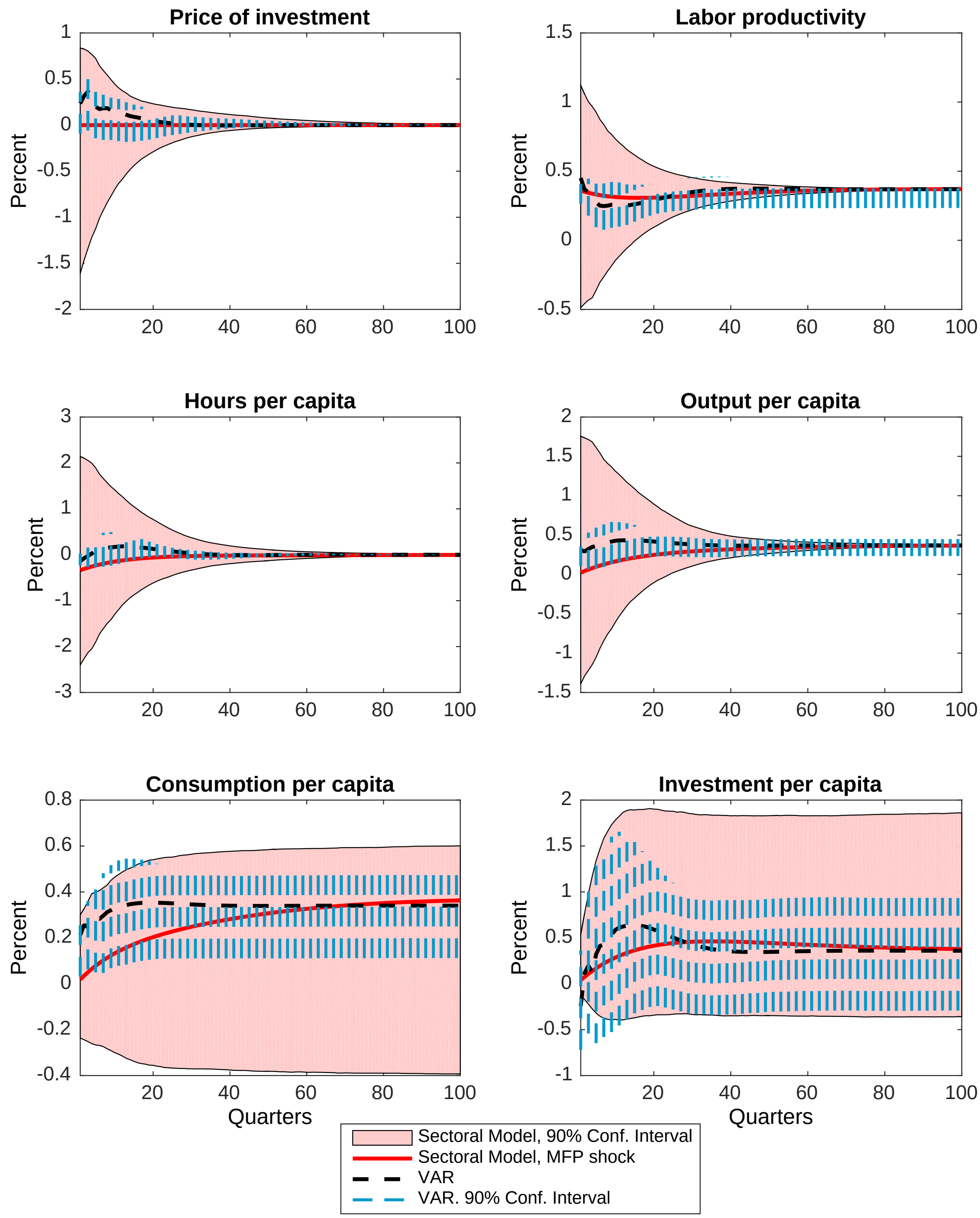
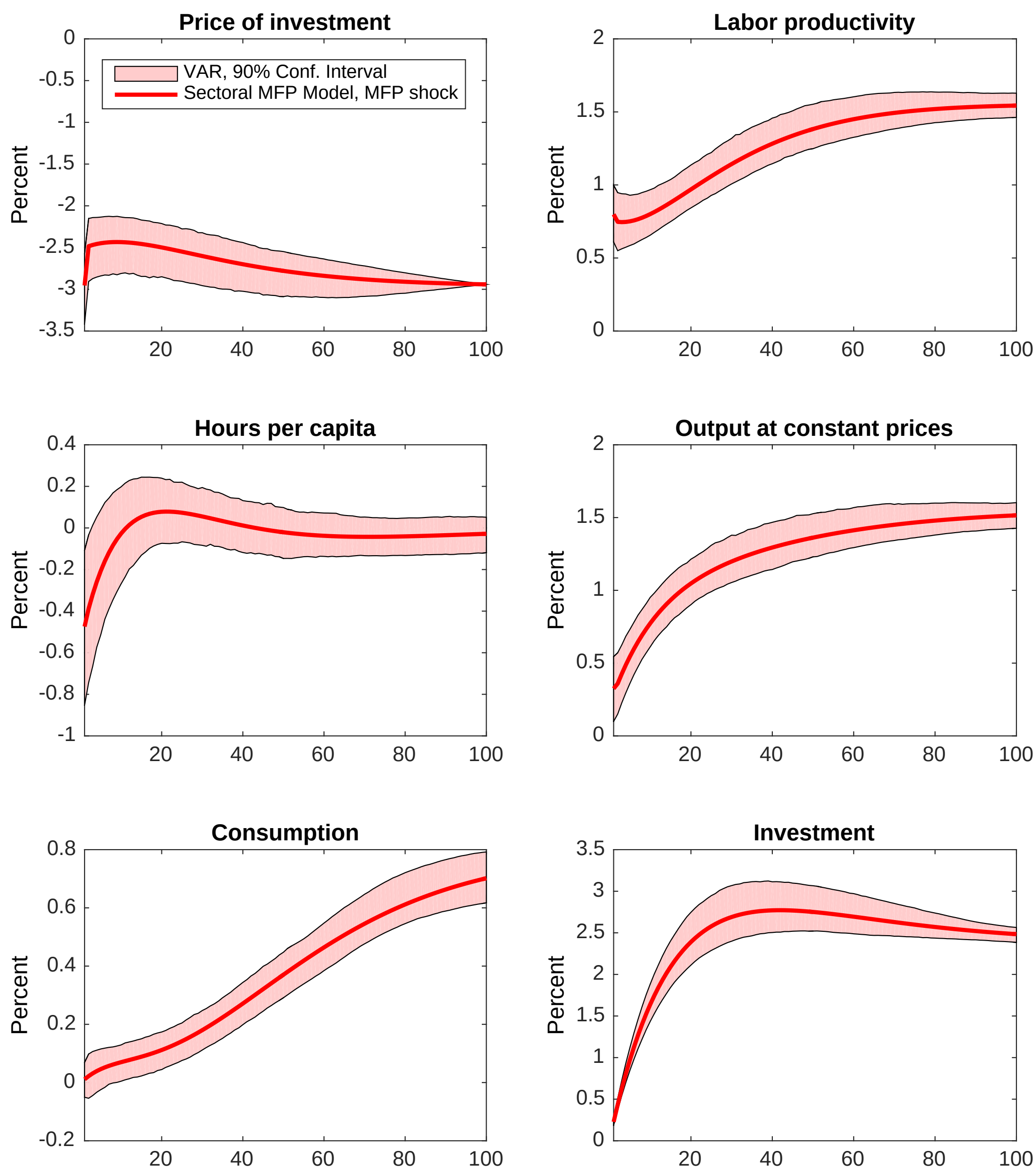
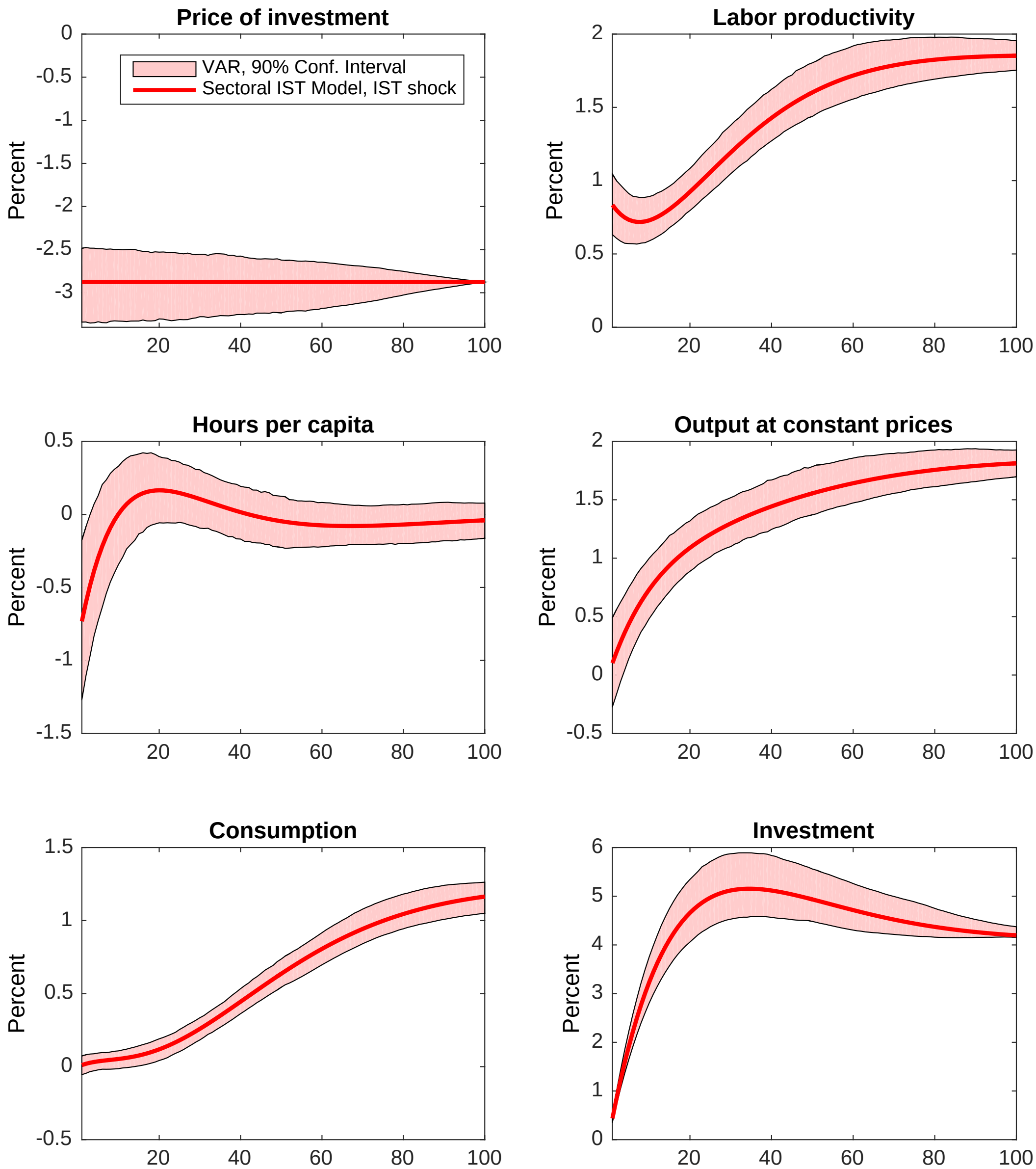


Figure 10: Responses to a Shock that Lowers the Relative Price of Investment Permanently, VAR Estimates Based on a Monte Carlo Experiment Using Large Samples Drawn from the Sectoral MFP Model



The responses shown are based on VARs from Monte Carlo experiment with 1000 samples each including 10,000 observations using the sectoral model as the data-generating process. The substantial narrowing of the confidence bands relative to the analogous experiment using samples including 100 observations (shown in Figure 6) and the true model responses being centered within the bands indicate that the model's sectoral MFP shock is recoverable by the VAR.

Figure 11: Responses to a Shock that Lowers the Relative Price of Investment Permanently, VAR Estimates Based on a Monte Carlo Experiment Using Large Samples Drawn from the Aggregate IST Model



The responses shown are based on VARs from Monte Carlo experiment with 1000 samples each including 10,000 observations using the aggregate IST model as the data-generating process. The substantial narrowing of the confidence bands relative to the analogous experiment using samples including 100 observations (shown in Figure 7) and the fact the true model responses to an being centered within the bands indicate that the model's IST shock is recoverable by the VAR.