Equations for New-keynesian model with government spending

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1. Model Equations

We sketch here the equations describing the equilibrium of the model. We drop the t subscript to denote the steady-state value of a particular variable.

$$
\begin{aligned}
c_t + k_t &= w_t n_t + (RR_t z_t + 1 - \delta) k_{t-1} + \left(1 - \frac{1}{X_t}\right) Y_t - \tau_t Y - b_t Y + R_{t-1} b_{t-1} Y && (1) \\
u_{c,t} &= \beta E_t \left(\frac{u_{c,t+1} R_t}{\pi_{t+1}}\right) && (2) \\
u_{c,t} v_t &= \beta E_t \left(u_{c,t+1} \left(RR_{t+1} + (1-\delta) v_{t+1}\right)\right) && (3) \\
u_{c,t} w_t &= u_{n,t} X_{wt} && (4) \\
Y_t &= A_t n_t^{1-\mu} (z_t k_{t-1})^{\mu} && (5) \\
(1-\mu) Y_t &= X_{pt} w_t n_t && (6) \\
\mu Y_t &= X_{pt} RR_t z_t k_{t-1} && (7) \\
\ln \pi_t - \iota_\pi \ln \pi_{t-1} &= \beta \left(E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t\right) - \varepsilon_\pi \ln (X_{pt}/X_p) + u_{p,t} && (8) \\
\omega_{c,t} - \iota_{wc} \ln \pi_{t-1} &= \beta \left(E_t \omega_{c,t+1} - \iota_{wc} \ln \pi_t\right) - \varepsilon_{wc} \ln (X_{wt}/X_{wc}) && (9) \\
R_t &= (R_{t-1})^{r_R} \pi_t^{r_\pi (1-r_R)} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{r_Y (1-r_R)} \overline{rr}^{1-r_R} && (10)
\end{aligned}
$$

Additional definitions given by

$$
\begin{aligned}
u_{c,t} &= \frac{1-\varepsilon_c}{1-\beta\varepsilon_c} \left(\frac{1}{c_t - \varepsilon_c c_{t-1}} - \frac{\beta \varepsilon_c}{c_{t+1} - \varepsilon_c c_t}\right) && (11) \\
u_{n,t} &= \tau n_t^{\eta} && (12)
\end{aligned}
$$

The equation for capacity is

$$
RR_t = \left(\frac{1}{\beta} - (1-\delta)\right) \left(\frac{\zeta}{1-\zeta} z_t + 1 - \frac{\zeta}{1-\zeta}\right) \tag{13}
$$

and the definition of investment is

$$
i_t = k_t - (1-\delta) k_{t-1} \tag{14}
$$

The optimality conditions related to investment are

$$
u_{c,t} v_t (1 - mac_t) = u_{ct} - \beta G_C u_{c,t+1} mac_{t+1} \tag{15}
$$

where:

$$
mac_t = \frac{d\phi_t}{di_t} = \phi (i_t - i_{t-1}) \tag{16}
$$

The remaining equations describe stochastic processes for the shocks and the evolution of government spending and the government budget constraint

$$
\begin{aligned}
g_t Y &= \left(1-\rho_g\right) gY + \rho_g g_{t-1} Y + \varepsilon_{gt} Y && (17)\\
g_t Y &= R_{t-1} b_{t-1} Y = \tau_t Y + b_t Y && (18)\\
\tau_t Y &= \rho_\tau \tau_{t-1} Y + (1-\rho_\tau)\left(\varepsilon_{\tau b} b_{t-1} Y + \varepsilon_{\tau g} g_t Y\right) && (19)
\end{aligned}
$$