Bayesian Estimation of a DSGE model

Why bother?

Practical problem: as the number of parameters to be estimated increases the likelihood might flatten out.

Bayesian approach brings in prior information to the problem, which can give us local identification where we had none before.

How does it work

Bayes' rule:

Let A and B be two events defined on a sample space, then

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)}$$

Now let X and Y be random variables

$$P_{x/y}(X/Y) = \frac{P_{y/x}(Y/X)P_x(X)}{P_y(Y)}$$

If Y is observed data and X is a collection of parameters, the $P_{y/x}(Y/X)$ is the likelihood. $P_x(X)$ is the prior and $P_{x/y}(X/Y)$ is the posterior.

Analytical Integration

If we could compute

$$\int_x P_{y/x}(Y/X)P_x(X)dx$$

Then we could also construct $P_y(Y)$, at that point we would know $P_{x/y}(X/Y)$.

This problem can be solved analytically for "judicious" choices of priors and models.

Analytical procedure generally not available for the estimation of DSGE models, since the parameters of interest are embedded in the reduced form of the model.

Numerical Integration

Can we save the day with numerical methods? We most certainly can.

The most widely used numerical method when estimating DSGE models is the Metropolis-Hastings algorithm.

It is one implementation of importance sampling:

- sample at random, but stick around a little longer where the posterior has a lot of mass

- still allow for escapes from areas of high mass

The MH Algorithm

Choose θ_0

Draw θ^* from $q(./\theta_i)$

Calculate ratio $r = \frac{P(Y/\theta^*)P(\theta*)}{P(Y/\theta_i)P(\theta_i)}$

If $r \geq 1$ then $\theta_{i+1} = \theta^*$ otherwise

$$\theta_{i+1} = \theta^* \text{with probability r}$$
$$\theta_{i+1} = \theta_i \text{with probability 1-r}$$

Draws from q constructed as above will converge to draws from the posterior if some regularity conditions are met. These conditions include that $q(./\theta)$ needs to cover the posterior distribution.