Solution to Problem Set 2

Question 1

Following the same decentralization as in Problem Set 1, the households' problem can be written as:

$$\begin{aligned}
\max_{c_t,i_t,l_t,k_t} \quad E_t & \quad \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \chi_0 \frac{(1-l_t)^{1-\chi)} - 1}{1-\chi} \right] \\
&+ \quad \beta^t \gamma_t \left[k_t - (1-\delta)k_{t-1} - q_t^{\frac{1-\alpha}{\alpha}} i_t \right] \\
&+ \quad \beta^t \lambda_t \left[r_t k_{t-1} + w_t l_t - c_t - i_t \right]
\end{aligned}$$

In addition to the two constraints, the FOCs from the households problem are:

$$\begin{aligned}
\frac{\partial}{\partial c_t} &= \frac{1}{c_t} - \lambda_t = 0 && (1) \\
\frac{\partial}{\partial l_t} &= -\chi_0(1-l_t)^{-\chi} + \lambda_t w_t = 0 && (2) \\
\frac{\partial}{\partial i_t} &= -q_t^{\frac{1-\alpha}{\alpha}} \gamma_t - \lambda_t = 0 && (3) \\
\frac{\partial}{\partial k_t} &= -(1-\delta)\beta\gamma_{t+1} + \gamma_t + \beta\lambda_{t+1} r_{t+1} = 0 && (4) \\
& && (5)
\end{aligned}$$

From the firms' profit maximization problem, given competitive markets for factor inputs and outputs, we have that:

$$\begin{aligned}
w_t &= (1-\alpha)\frac{y_t}{l_t} && (6) \\
r_t &= \alpha \frac{y_t}{k_{t-1}} && (7) \\
y_t &= k_{t-1}^{\alpha} (z_t l_t)^{1-\alpha} && (8)
\end{aligned}$$

Finally, the list of necessary conditions for an equilibrium includes the processes governing q_t and z_t, which we posit are governed by AR(1) processes expressed in growth rates.

To express the necessary conditions for an equilibrium in terms of stationary variables, consider the following transformations:

$$\begin{array}{ccc} \tilde{c}_t = \frac{c_t}{z_t q_t} & \tilde{i}_t = \frac{i}{z_t q_t} & \tilde{\lambda}_t = \lambda_t z_t q_t \\ \tilde{\gamma}_t = \gamma_t z_t q_t^{\frac{1}{\alpha}} & \tilde{k}_t = \frac{k_t}{z_t q_t^1 \alpha} & \tilde{y}_t = \frac{y_t}{z_t q_t} \\ \tilde{w}_t = \frac{w_t}{z_t q_t} & \tilde{r}_t = \frac{r_t}{q_t^{1-\frac{1}{\alpha}}} & \tilde{l}_t = l_t \end{array} \tag{9}$$

Applying these transformations, one obtains:

$$\frac{1}{\tilde{c}_t} - \tilde{\lambda}_t = 0 \tag{10}$$

$$-\chi_0(1-l_t)^{-\chi} + \tilde{\lambda}_t \tilde{w}_t = 0 \tag{11}$$

$$-\tilde{\gamma}_t - \tilde{\lambda}_t = 0 \tag{12}$$

$$-(1-\delta)\beta\tilde{\gamma}_{t+1} + \tilde{\gamma}_t \mu_{z,t+1}\mu_{q,t+1}^{\frac{1}{\alpha}} + \beta\tilde{\lambda}_{t+1}\tilde{r}_{t+1} = 0 \tag{13}$$

$$\tilde{k}_t = (1-\delta)\frac{1}{\mu_{z,t}\mu_{q,t}^{\frac{1}{\alpha}}}\tilde{k}_{t-1} + \tilde{i}_t \tag{14}$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{i}_t \tag{15}$$

$$\tilde{w}_t = (1-\alpha)\frac{\tilde{y}_t}{l_t} \tag{16}$$

$$\tilde{r}_t = \alpha\frac{\tilde{y}_t}{\tilde{k}_{t-1}}\mu_{z,t}\mu_{q,t}^{\frac{1}{\alpha}} \tag{17}$$

$$\tilde{y}_t = \tilde{k}_t^{\alpha} l_t^{1-\alpha}\left(\frac{1}{\mu_{z,t}}\frac{1}{\mu_{q,t}^{\frac{1}{\alpha}}}\right)^{\alpha} \tag{18}$$

To find the steady state restrictions consider the following strategy. First from the equation for capital accumulation, you can see that

$$\tilde{k} = \frac{1}{1-\frac{(1-\delta)}{\mu_z \mu_q^{\frac{1}{\alpha}}}}\tilde{i}$$

or equivalently that

$$S \equiv \frac{\tilde{i}}{\tilde{y}} = \left(1 - \frac{(1-\delta)}{\mu_z \mu_q^{\frac{1}{\alpha}}}\right)\frac{\tilde{k}}{\tilde{y}}$$

From the firms' demand for capital, we also know that

$$r = \alpha \frac{\tilde{y}}{\tilde{k}} \mu_z \mu_q^{\frac{1}{\alpha}}$$

But from households' FOC with respect to k_t, we also know that

$$r = \frac{\mu_z \mu_q^{\frac{1}{\alpha}} - (1-\delta)\beta}{\beta}$$

Combining the last three equations, we can solve for S in terms of parameters only:

$$S = \frac{\alpha}{r} * (\mu_z * \mu_q^{\frac{1}{\alpha}} - 1 + \delta) \tag{19}$$

Next, from the households' FOC with respect to labor, we can see that:

$$\chi_0 (1-l)^{-\chi} = \tilde{\lambda} \tilde{w}$$

But from the FOC with respect to consumption and from the firms' labor demand equation:

$$\chi_0 (1-l)^{-\chi} = \frac{1}{\tilde{c}} (1-\alpha) \frac{\tilde{y}}{l}$$

Using the resource constraint $\tilde{y}_t = \tilde{c}_t + \tilde{i}_t$, we can express the equation above in terms of the steady-state savings rate:

$$\chi_0 = \frac{(1-l)^{\chi}}{1-S} \frac{(1-\alpha)}{l}$$

Which yields the restriction for χ_0 in terms of parameters and the choice for the labor supply in steady state.

The program *call_rblabtrend.m* compares the response to a unit-root MFP shock in a model with steady state growth and in a model without steady-state growth. The program *call_mfpVSist.m*— compares the response to a unit root MFP shock against the response a unit root shock to the investment technology. Both shocks are sized so as to raise output by 1 percent in the long run.

Question 2 The FOC from the firms' profit maximization problem is:

$$\begin{aligned} \frac{\partial}{\partial P_t(f)} &= (1+\tau_p) y_t(f)(1-\phi_t) + (P_t(f)(1+\tau_p) - \Sigma_t) \frac{\partial y_t(f)}{\partial P_t(f)} (1-\phi_t) \\ - \quad & (P_t(f)(1+\tau_p) - \Sigma_t) y_t(f) \frac{\partial \phi_t}{\partial P_t(f)} - \beta (E_t P_{t+1}(f)(1+\tau_p) - E_t \Sigma_{t+1}) y_{t+1}(f) E_t \frac{\partial \phi_{t+1}}{\partial P_t(f)} = 0 \end{aligned}$$

Notice that:

$$\frac{\partial y_t(f)}{\partial P_t(f)} = -\frac{1+\theta_p}{\theta_p}\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}-1}\frac{y_t}{P_t}$$
$$\frac{\partial \phi_t}{\partial P_t(f)} = \phi_1\left(\frac{P_t(f)}{\pi P_{t-1}(f)}-1\right)\frac{1}{\pi P_{t-1}(f)}$$
$$\frac{\partial \phi_{t+1}}{\partial P_t(f)} = -\phi_1\left(\frac{P_{t+1}(f)}{\pi P_t(f)}-1\right)\frac{P_{t+1}(f)}{\pi P_t(f)^2}$$

Also notice that in equilibrium all firms set the same prices, therefore $P_t(f) = P_t$ and $y_t(f) = y_t$. Consequently, the first-order condition for profit maximization simplifies to:

$$\frac{\partial}{\partial P_t(f)} = (1+\tau_p)(1-\phi_t) - ((1+\tau_p)-\sigma_t)\frac{1+\theta_p}{\theta_p}(1-\phi_t)$$
$$- \quad ((1+\tau_p)-\sigma_t)\phi_1\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi} + \beta((1+\tau_p)-E_t\sigma_{t+1})\frac{y_{t+1}}{y_t}\phi_1\left(\frac{E_t\pi_{t+1}}{\pi}-1\right)\frac{E_t\pi_{t+1}^2}{\pi} = 0$$

where $\sigma_t = \frac{\Sigma_t}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$. Linearizing around a steady state with zero inflation (i.e. $\pi = 1$), as in the case for Calvo contracts described in Handout 5, one obtains:

$$\frac{1+\theta_p}{\theta_p}\hat{\sigma}_t - \phi_1\tau_p\hat{\pi}_t + \phi_1\tau_p\beta E_t\hat{\pi}_{t+1} = 0$$

which can be rearranged as:

$$\pi_t = \beta E_t\hat{\pi}_{t+1} + \frac{1}{\phi_1}\frac{1+\theta_p}{\theta_p\tau_p}\hat{\sigma}_t$$

As shown in handout 5, with Calvo contracts, we have instead:

$$\pi_t = \beta E_t\hat{\pi}_{t+1} + \kappa_p\hat{\sigma}_t$$

But then the two equations can be made equivalent to first order by setting ϕ_1 so that:

$$\frac{1}{\phi_1}\frac{1+\theta_p}{\theta_p\tau_p} = \kappa_p$$

or

$$\phi_1 = \frac{1+\theta_p}{\theta_p\tau_p\kappa_p}$$

Question 3

See the matlab program *call_stickyprotemberg.m* and related Dynare file *stickyprotemberg.mod*. As usual, remember to update the path in *setpathdynare4.m* to point to the directory where you installed Dynare in your local host.